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The Assessment of the Uncertainty of Updated Stand-Level Inventory Data

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Predictions of growth and yield are essential in forest management planning. Growth predictions are usually obtained by applying complex simulation systems, whose accuracy is difficult to assess. Moreover, the computerised updating of old inventory data is increasing in the management of forest planning systems. A common characteristic of prediction models is that the uncertainties involved are usually not considered in the decision-making process. In this paper, two methods for assessing the uncertainty of updated forest inventory data were studied. The considered methods were (i) the models of observed errors and (ii) the k-nearest neighbour method. The derived assessments of uncertainty were compared with the empirical estimates of uncertainty. The practical utilisation of both methods was considered as well. The uncertainty assessments of updated stand-level inventory data using both methods were found to be feasible. The main advantages of the two studied methods include that bias as well as accuracy can be assessed.

Keywords measurement error, non-parametric methods, observed error, simulation, stand-level inventory, uncertainty
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1 Introduction

Field data concerning the state of the forests, as well as predictions of forest growth and yield, are essential information sources in forest management planning. Inventory data are used in forest planning systems to predict stand growth under different management schedules and to optimise the management schedules for the stands depending on the landowner's preferences or objectives (e.g. Leskinen 2001) and regulations, as well as the recommendations based on current forest management practices. Forest planning systems are usually quite complex, containing models for predicting the development of stands, e.g. models for regeneration, growth and mortality, and models for simulating the impact of different management schedules based on these predictions (e.g. Jonsson et al. 1993, Siitonen 1993, Lundström and Söderberg 1996, Eid and Hobbelstad 2000). Accurate predictions of stand growth and yield are essential, because inaccurate predictions lead to wrong conclusions and non-optimal treatment schedules (Kangas and Kangas 1997).

The practice in the Nordic Countries is for field data to be usually collected stand-by-stand using subjective forest inventory methods, e.g. ocular inventory methods. In Finland, for example, the inventory data on non-industrial private forests are mainly collected by using Bitterlich (1984) sample plots. The stand basal area is assessed as the average of the subjectively selected representative sample plots. Furthermore, measurers often change the average basal area of the sample plots based on their own subjective views. Moreover, tree heights and diameters at breast height are not measured on the sample plots; instead, the trees are tallied using a relascope and a subjectively chosen basal area median diameter tree per tree species per stand is assessed by the measurer. Thus, it is difficult to estimate the sampling errors and the accuracy of stand-level inventory.

The accuracy of stand-level inventory contains wide variation due to the subjectivity of the method. For example, the measurement error of the basal area (BA) varies from about 10% to 22% (e.g. Mähönen 1984, Laasasenaho and Päivinen 1986, Ståhl 1992, Pigg 1994, Haara and Korhonen 2004). In addition, the mean diameter at breast height (D_{gM}), the mean height (H_{gM}), and the mean age (Age) in stand-level inventory are determined by referring to the same basal-area median-diameter tree. This implies, for example, that errors in these stand characteristics are positively correlated (e.g. Ståhl 1992, Pigg 1994, Haara 2003). The measurement error for mean age is about 20% and for mean diameter and mean height about 10–20% (e.g. Mähönen 1984, Laasasenaho and Päivinen 1986, Ståhl 1992, Pigg 1994, Haara and Korhonen 2004). In stand-level inventory, the standard error of the mean stand volume (V) can vary between 15% and 38% (e.g. Poso 1983, Laasasenaho and Päivinen 1986, Ståhl 1992, Haara and Korhonen 2004). The considerable variation between measurers' accuracy has been also noted in many studies (e.g. Laasasenaho and Päivinen 1986, Nersten and Næsset 1992, Ståhl 1992, Kangas et al. 2002, Haara and Korhonen 2004).

Long-term management planning typically uses the diameter distribution approach in growth and yield predictions (e.g. Clutter et al. 1983, Siitonen 1993). When doing so, the collected field data with tree and stand characteristics are used to estimate the theoretical diameter distribution of trees. The practice in Finland is to use basal area diameter distributions instead of the stem diameter distributions (e.g. Kilkki et al. 1989, Maltamo 1998). Each sample tree from the theoretical diameter distribution predicts a certain number of trees in the stand. The mean stand volume can be predicted by multiplying the volume of each sample tree by the prediction of its number and by summing up the estimated volumes.

The uncertainty of growth and yield predictions should be taken into account in forest planning and decision-making (e.g. Hof et al. 1988, Hof and Pickens 1991, Pickens et al. 1991, Hof et al. 1992, Kangas 1999). There are several approaches to assessing this uncertainty. Perhaps the simplest way to assess the uncertainty of growth predictions is to use re-measured data and compare the growth predictions obtained to observed growth. However, empirical assessments are time and place constrained, i.e. the assessments carried out in a certain time-and-place combination can not be directly applied elsewhere (Kangas 1999). Besides, the assessments of the reference growth data are usually based on accurate measurements instead of ocular assessments. The second approach to assessing the accuracy of growth predictions is to utilise the Monte Carlo simulation or Taylor series approximation methods (e.g. Gertner and Dzialowy 1984, Mowrer and Frayer 1986, Gertner 1987, Mowrer 1991, Gertner et al. 1995, Kangas 1997). When using these methods, the total prediction error is composed of several error sources (Kangas 1999). In both methods, accuracy can be assessed without independent re-measurement of data sets. However, from the technical point of view, it may be very difficult to take all error sources into account, especially when dealing with large model sets. A third possibility in studying the accuracy of simulation systems is to model the observed (past) errors as functions of explanatory variables at the aggregated level (e.g. Hansen and Hahn 1983, Soares et al. 1995, Kangas 1999). In this approach, the errors caused by different growth and yield models need not be specified. The main restriction in using this approach is the requirement of independent continuous inventory data, in which both planning data and correct data are available at the beginning and at the end of the planning period. Comparative studies between different methods for the purpose of determining their accuracy, e.g. Kangas (1999), are rare. Most studies have focused on dealing with Monte Carlo simulation or Taylor series approximation methods.

In addition to the above techniques, Haara (2002) assessed the uncertainty of growth and yield predictions by using the k-nearest neighbour method (*k*-NN method). The uncertainty of the predicted stand characteristics of the target stand was derived from the uncertainty assessments of growth predictions of the nearest neighbour stands. The variables of the distance function, as well as the weights of these variables, were chosen using multi-objective optimisation (Haara 2002). The differences between the predicted growth of the target stand and the predicted growths of the reference stands were minimised. As a result, the stand characteristics and predicted growth were as similar as possible between the target stand and the reference stands. In practice, the quality of the *k*-NN method predictions depends on the availability of extensive reference data. If good reference data are available, the examined method is a very promising way to predict the uncertainty of growth predictions (Haara 2002).

The quality of forest management planning depends greatly on the accuracy of the inventory data, which can be improved, but doing so results in increasing inventory costs. In general, the costs of field inventory are significant, comprising approximately half of the total costs of forest management planning (Uuttera et al. 2002). On the other hand, the costs of the field inventory can be reduced by utilizing old inventory data by updating the stand characteristics contained in the old data computationally using forest simulation systems (e.g. Clutter et al. 1983, Siitonen 1993). However, the accuracy of the updated data must be adequate for planning purposes. Computational updating is already being used in forest planning without accurate knowledge of the uncertainty of the updated stand characteristics, although there are some studies, which have demonstrated that updated stand characteristics can be as accurate as stand characteristics obtained from new inventory data (e.g. Pussinen 1992, Anttila 2002, Hyvönen and Korhonen 2003)

The aim of this paper is to further study the uncertainty of updated stand-level inventory data by comparing two different methods for assessing the uncertainty. First, the uncertainty of growth and yield predictions is estimated by modelling the observed errors obtained by comparing the empirical stand characteristics with the updated stand characteristics. Second, the assessment of uncertainty is carried out by applying the *k*-nearest neighbour (*k*-NN) method and multi-objective optimisation. These two assessment methods have been presented as promising methods for predicting uncertainty, and in this study the methods are further studied and compared in connection with a large amount of test data. The updating is carried out using the MELA forest management planning system (Redsven et al. 2004). The assessments of uncertainty are compared with the empirical uncertainty. The sources of uncertainty considered in this study are the errors in the basic forest inventory data and the errors of the predictions of the forest development due to these inventory errors.

2 Material and Methods

2.1 Study Material

The study data consisted of three independent data sets. The first data set was obtained from fixed-radius permanent plots (INKA data) located in pure and mixed stands (Gustavsen et al. 1988). The INKA data were collected between 1976 and 1992 by Finnish Forest Research Institute staff from forest stands growing on mineral soils (Table 1). Three fixed circular sample plots were measured in each stand 1–3 times at intervals of 5 years. Plots were located 40 meters apart from each other. The plot size varied: at least 120 sample trees per stand were measured in Southern Finland and 100 sample trees per stand in Northern Finland. The area of the study plots varied between 0.008 and 0.13 hectares mean area being 0.011 hectares. Tree diameters at breast height of all trees of the plot were measured but tree heights were measured only from trees, which were located in a smaller plot at the center of each sample plot. The area of smaller plot was 1/3 of the area of the sample plot. The INKA data consisted of a total of 754 stands.

The second data set was comprised of three independent data sets collected compartment-by-compartment from Eastern Finland in 1998–2002 (Hyvönen 2002, Hyvönen and Korhonen 2003, Haara and Korhonen 2004). These three data sets were combined and consisted of a total of 1223 stands (Table 1). A stand-level inventory was first carried out by measurers in large stand data. The accuracy of the stand-level inventory was then controlled by measuring a large check inventory data. A systematic network of fixed circular sample plots was measured within each stand of the checking inventory data. This dataset is referred as CONTROL data. The number of the plots and the radius of the plots depended on the area of each stand, on the development class of the particular stand, and on the number of tree species within the stand. The average radius of the plots was 7.5 metres, and the radius of the plots varied from 3.99 metres (young stands) to 10 metres (mature stands). The average number of sample plots was 6.2 plots per stand. The tree species and the diameter at breast height were determined for each tree on the plots. The heights of the basal-area median-diameter trees of all tree species were measured on at least three plots within each stand.

The third data set (NORTH data) consisted of independent and controlled compartment data including 1842 stands from Northern Finland and collected in 1990–1994 (Haara 2003). A stand-level inventory was carried out by measurers. The control inventory was then done by measuring a systematic net of relascope sample plots in each stand. The average number of sample plots was

Table 1. The main stand characteristics of the INKA data, CONTROL data, and NORTH data.

	Min	Max	Mean	SD
INKA (754 stands)				
D_{gM} (cm)	1.9	35.2	15.8	6.3
H_{gM} (m)	1.9	28.7	12.5	5.6
BA ($m^2 ha^{-1}$)	0.2	38.0	16.4	8.4
V ($m^3 ha^{-1}$)	0.7	387.8	114.5	84.0
Age (years)	2	154	52.7	30.8
CONTROL (1223 stands)				
D_{gM} (cm)	7.5	42.6	19.2	5.9
H_{gM} (m)	4.3	31.9	15.5	5.2
BA ($m^2 ha^{-1}$)	1.5	40.8	20.3	6.4
V ($m^3 ha^{-1}$)	8.5	447.4	155.8	80.0
Age (years)	6	179	56.9	26.4
NORTH (1842 stands)				
D_{gM} (cm)	1	31.6	15.1	5.5
H_{gM} (m)	1.3	16.6	9.7	3.1
BA ($m^2 ha^{-1}$)	1	36	10.8	6.9
V ($m^3 ha^{-1}$)	1	250	60.3	46.7

Table 2. The study data sets.

Data	Origin	Format	Errors	Purpose
INKA		Treewise	None	Observed errors of growth predictions
CONTROL		Standwise	From measurers	Measurement errors, error generation
NORTH		Standwise	From measurers	Measurement errors, error generation
INKA1	INKA	Standwise	Diameter distribution model errors (DDME)	Test data for uncertainty assessment methods, original data for simulation of standwise data with measurement errors
CONTROL1	INKA1	Standwise	Simulated from CONTROL+DDME	Modelling data for the models of observed errors, reference data for the *k*-NN method and multi-objective optimisation
CONTROL2	INKA1	Standwise	Simulated from CONTROL+DDME	Test data for uncertainty assessment methods
NORTH2	INKA1	Standwise	Simulated from NORTH+DDME	Test data for uncertainty assessment methods

7.7 plots per stand. Tree species and diameters at breast height of all trees on the sample plots were determined (Table 1). The heights of the basal-area median-diameter trees of all tree species within the plot were measured. The three datasets, and the four data sets generated from those in Chapters 2.2 and 2.3, are introduced in Table 2.

2.2 Generation of True Stand-Level Inventory Data

Tree-specific stand data were first generated from the INKA data. The trees from three plots in each stand were combined to represent the empirical diameter distribution of the stand. The heights of the trees were taken only from the sample trees. Any missing heights h were estimated for each tree using Näslund's (1936) height model

$$h = 1.3 + \frac{d^2}{\left(b_0 + b_1 d\right)^2} + e \qquad (1)$$

where d refers to the diameter. The parameters b_0, b_1 and Var(e) were estimated separately for each stand. The missing tree heights were predicted using the model (1) so that a random variable sampled from the normal distribution N(0, Var(e)) was added to the predictions. The volumes of the trees from the diameter distributions were calculated using Laasasenaho's (1982) specieswise volume models. The correct stand volumes were obtained by summing up these tree volumes.

True stand-level inventory data including diameter distribution model errors, referred as INKA1, were generated from the true tree-specific stand data as outlined above (Fig. 1). The structure of the INKA1 data was the same as in current forest planning practice in Finland (Koivuniemi and Korhonen 2006, Metsäsuunnittelun maastotyöopas 2006). Each tree species was described by the basal area, the diameter at breast height of the basal-area median tree, the height of the basal-area median tree, and the age of the basal-area median tree. These species-specific characteristics were calculated from empirical diameter distributions. The theoretical basal area diameter distributions were predicted from the calculated stand characteristics using the three-parametric approach of the Weibull function (Mykkänen 1986, Kilkki et al. 1989). The heights of the sample trees obtained from the theoretical diameter distribution were predicted using Veltheim's (1987) height models. Then the predicted heights were calibrated with ratio estimation using the height of the basal-area median tree of the stand. The volumes of the model trees obtained from the theoretical diameter distribution were estimated using Laasasenaho's (1982) volume models. The volume estimates of the stand-level inventory data were obtained by summing up these volumes. Furthermore, the basal area median tree of the

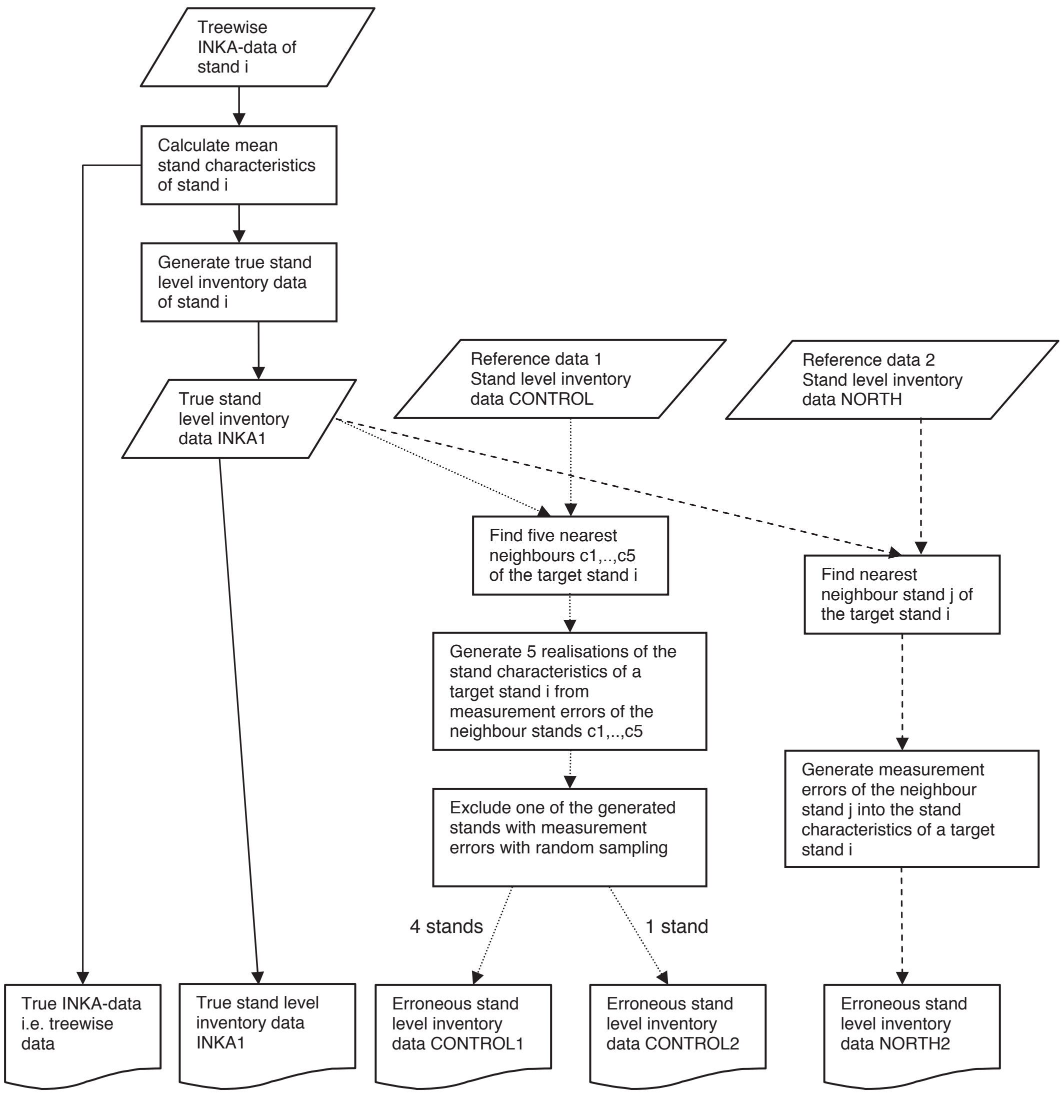

Fig. 1. Flow chart showing the generation of the study data.

stand was obtained from the theoretical diameter distribution. The diameter, height and age of the basal area median tree of the stand were used as stand's D_{gM}, H_{gM} and Age.

2.3 Generation of Erroneous Stand-Level Inventory Data

In order to assess the uncertainty of growth and yield predictions by examining practical stand-level inventory data, the study data should also include measurement errors. Haara (2003) found the one nearest neighbour method (1nn method) to be a useful tool for generating error structures for target stands reflecting those in stand-level inventory. In the method, the measurement errors of the neighbour stand are used directly as the measurement errors of the target stand. The differences between the stand characteristics (e.g. main tree species, stand basal area) of the target stand and neighboring stand are as small as possible. In this study,

Table 3. The root mean square errors (RMSE) and biases of the assessed stand characteristics in the two control inventory data (CONTROL and NORTH). The relative RMSEs and biases are shown in parentheses.

	CONTROL		NORTH	
	RMSE	Bias	RMSE	Bias
D_{gM} (cm)	2.5 (13.1)	0.6 (3.0)	2.5 (13.1)	0.6 (3.0)
H_{gM} (m)	2.4 (15.5)	0.05 (0.3)	2.4 (15.5)	0.05 (0.3)
BA ($m^2 ha^{-1}$)	3.9 (19.3)	0.6 (3.1)	3.9 (19.3)	0.6 (3.1)
V ($m^3 ha^{-1}$)	38.6 (24.8)	4.0 (2.6)	38.6 (24.8)	4.0 (2.6)

k error realisations of the stand characteristics of the target stand are needed and the errors of the *k* nearest neighbours are used one by one, i.e. the *k*-NN method is applied. Because the use of the *k*-NN method requires extensive reference data, the NORTH and the CONTROL data sets were utilised to generate the measurement errors of the stand-level inventory into the INKA1 data (Fig. 1). The procedure was as follows. First, the measurement errors of both NORTH and CONTROL data sets were examined by deriving true tree and stand characteristics. The missing heights of the sample trees were estimated using Veltheim's (1987) height models. The height predictions were calibrated using the heights of the stand's sample trees. The trees in the sample plots were combined to provide a compounded empirical stand diameter distribution. This distribution was used to calculate the true stand characteristics. Errors in the true stand characteristics originated from the errors in the measurement of the sample trees, from the height and volume models, and from the sampling error in the control inventory. The measurement errors of the stand-level inventory of CONTROL data and NORTH data were calculated by comparing true stand characteristics and assessed stand characteristics (Table 3). When the sampling error of the checking inventory was noted, the RMSE of the stand volume of CONTROL data decreased 3.4 percent units, the RMSE of the stand basal area 3.6 percent units, and RMSEs of the D_{gM} and the H_{gM} decreased 2.2 and 0.7 percent units, respectively.

After studying the measurement errors of the CONTROL data and the NORTH data, the next step was to use them as the reference data for the *k*-NN method for generating erroneous stand-level data in the 1st and 2nd measurements of the INKA data. Stand-level data with measurement errors were first generated from the measurement errors of the CONTROL data. The level and the structure of the measurement errors were assumed to correlate with the stand characteristics. The search for five nearest-neighbour stands was done by using commonly assessed stand characteristics as the distance function variables in the *k*-NN method. Similarly, distance functions were applied depending on the tree species. The chosen standardised variables of the distance function were the basal area median diameter, the stand mean basal area, and the proportion of tree species in the stand in terms of the basal area per hectare. Five different error realisations for the target stand were obtained from the measurement errors of the target stand's five neighbour stands. One of the five random error generations of each target stand was excluded from the data using simple random sampling. The remaining data with four measurement error realisations of each stand (CONTROL1) were used for modelling the observed errors and the excluded data with one measurement error realisation for each stand were used as the test data (CONTROL 2). In this way, each stand had different measurement errors in the modelling and in the testing phase. The second test data (NORTH2) were generated with the 1nn method and with NORTH data as the reference data.

The generated stand level data CONTROL1 were used as modelling data for the models of the observed errors as well as for the reference data for the *k*-NN method for the uncertainty assessments of the updated stand level data. CONTROL2 and NORTH2 data were used for testing both of these

Table 4. The RMSEs and biases of the stand characteristics in the true INKA stand data including model errors (INKA1) and reference INKA stand data including measurement errors generated from the CONTROL data (CONTROL1).

	1st measurement		2nd measurement	
	RMSE	Bias	RMSE	Bias
INKA1				
DgM (cm)	0.8 (4.9)	–0.1 (–0.4)	0.8 (4.6)	–0.03 (–0.2)
HgM (m)	1.3 (11.0)	0.05 (0.4)	1.1 (8.6)	–0.03 (–0.3)
BA (m^2 ha^{-1})	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
V (m^3 ha^{-1})	6.2 (5.6)	–1.0 (–0.9)	8.6 (6.5)	–2.2 (–1.7)
CONTROL1				
DgM (cm)	2.0 (12.8)	0.4 (2.4)	2.1 (12.7)	0.5 (2.7)
HgM (m)	2.3 (18.8)	0.04 (0.3)	2.3 (17.1)	–0.02 (–0.1)
BA (m^2 ha^{-1})	2.9 (18.0)	0.0 (0.0)	3.2 (16.9)	0.05 (0.2)
V (m^3 ha^{-1})	28.0 (25.0)	–0.4 (–0.4)	32.8 (24.5)	–1.2 (–0.9)

Table 5. The RMSEs and biases of the two test data. In the first test data (CONTROL2) the measurement errors in the stand level inventory are generated from the CONTROL data and in the second test data (NORTH2) the measurement errors are generated from NORTH data.

	1st measurement		2nd measurement	
	RMSE	Bias	RMSE	Bias
CONTROL2				
D_{gM} (cm)	2.1 (13.5)	0.3 (2.1)	2.1 (12.4)	0.3 (2.0)
H_{gM} (m)	2.3 (18.7)	0.1 (0.9)	2.3 (17.2)	0.01 (0.1)
BA (m^2 ha^{-1})	2.9 (18.2)	0.2 (1.0)	3.5 (18.5)	0.3 (1.4)
V (m^3 ha^{-1})	26.8 (24.4)	1.4 (1.3)	35.6 (26.0)	1.2 (0.9)
NORTH2				
D_{gM} (cm)				
H_{gM} (m)	1.9 (12.3)	–0.03 (–0.2)	2.0 (12.0)	0.1 (0.4)
BA (m^2 ha^{-1})	2.4 (20.0)	0.1 (0.9)	18.4 (2.4)	0.1 (0.7)
V (m^3 ha^{-1})	2.5 (15.6)	0.3 (1.7)	2.7 (14.0)	0.4 (1.9)

methods. The errors in the stand characteristics in the data including diameter distribution model errors (INKA1) as well as the errors in the modelling data (CONTROL1) are presented in Table 4, and the errors in the stand characteristics in both test data are presented in Table 5.

2.4 Growth Simulations

All stand-level inventory data (i.e. correct stand inventory data, modelling data and both test data) were updated by using the MELA forest simulator (Siitonen 1993, Hynynen et al. 2002, Redsven et al. 2004) for modelling growth, regeneration establishment, and tree mortality. The growth of the 1st measurements of the correct stand inventory data and modelling data were predicted 5 and 10 years, and the growth of the 2nd measurements were predicted 5 years. The growth of both test data (CONTROL2, NORTH2) was predicted 5 and 10 years. The logging and silvicultural operations carried out between inventories were simulated. The timing of these operations had been recorded in the inventory. The cuttings were simulated following the thinning and regeneration models

(Hyvän metsänhoidon… 2001). The uncertainty of the growth predictions of the MELA forest simulator included errors of the growth, regeneration and mortality models, and the errors due to the processing of the inventory data besides the measurement errors of the inventory data.

2.5 Models for Observed Errors

The first method for assessing the uncertainty of the updated stand characteristics was to model the observed errors. The empirical errors in the stand characteristics were achieved by calculating the differences between the updated stand characteristics and empirical stand characteristics. The models for the observed errors were estimated for both true and erroneous stand-level data. In both cases, the models were estimated first for stands in which no logging or silvicultural treatments had been carried out during the simulation period, and secondly, for the entire data. The models for the treated stands were not estimated because of the small number of these stands. The models for the observed errors of the basal area median diameter, the basal area median height, the basal area, and the mean stand volume were estimated. The models were of the form:

$$\ln(\mathrm{Error}_i^2) = \alpha + \beta_1 SC_1 + \beta_2 SC_2 + ... + e_i \qquad (2)$$

or

$$\mathrm{Error}_i^2 = \alpha + \beta_1 SC_1 + \beta_2 SC_2 + ... + e_i \qquad (3)$$

where Error_i^2 denotes the squared observed error of the updated stand characteristics, SC_k denotes stand and site characteristics (k = 1,2,…), and e_i is an error term.

The stand and site characteristics, which could have been measured as such or derived from stand level inventory data (e.g. growth predictions), were used as independent variables in these models. By using these models, the predictions of stand's MSEs of the updated stand characteristics can be achieved. Furthermore, the bias models for the observed errors were estimated. In these models, the observed errors of the updated stand characteristics, i.e. dependent variables in these models, were not squared.

2.6 K-nearest Neighbour Method and Multi-Objective Optimisation

The second method for assessing the uncertainty of the updated stand characteristics was the combination of the *k*-NN method (Härdle 1989, Altman 1992) and multi-objective optimisation. The uncertainty of the predicted stand characteristics of the target stand, i.e. RMSEs and biases, was predicted from the uncertainty of the growth predictions of the 10 nearest neighbour stands. The search for the reference stands was carried out by using standardised commonly measured stand characteristics, i.e. stand basal area, stand age and the class variable main tree species, as the variables of the distance function. The variables of the similarity distance function, as well as the weights of the variables, were determined using multi-objective optimisation. The non-linear programming algorithm (Hooke and Jeeves 1961) was used to find the combination of decision variables minimizing the objective function. The computer program developed by Osyczka (1984) was modified and adapted to deal with the *k*-NN method. The differences between the predicted growth of the target stand and the predicted growth of the reference stand were minimised in the optimisation. Thus, the stand characteristics and predicted growth were as similar as possible between the target stand and the reference stands. The (R)MSEs and biases of the updated stand characteristics of the neighbour stands were used as target stand's ucertainty assessments.

In the case of the INKA1 data, i.e. true stand-level inventory data, the reference data of the target stand consisted of all stands besides target stand in INKA1 data. As regards the stand data with measurement errors, the reference data consisted of the CONTROL1 data besides target stand, i.e. INKA data with measurements errors generated from the CONTROL data.

2.7 Uncertainty Assessments of the Updated Stand Characteristics

The usability of both assessment methods was tested by using the true stand-level inventory data INKA1 and two generated test data, i.e. CONTROL2 and NORTH2. All of these data sets were updated and the empirical errors of the growth predictions were calculated from the differences between the updated stand characteristics and the true stand characteristics. The test criteria used in the comparison of the predictions were RMSE and bias of the observed errors. The RMSE and the bias of the empirical errors were calculated as

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2}{n-1}} \tag{4}$$

and

$$\mathrm{bias} = \frac{1}{n}\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right) \tag{5}$$

where Y_i denotes the true value of the stand characteristics, $\hat{Y}_i$ denotes the predicted value of the stand characteristics, and n is the number of stands. The relative RMSE and the relative bias are obtained by dividing RMSE and bias by the average of the true stand characteristics.

The uncertainty of the updated stand characteristics for the true stand-level inventory data and two generated test stand data were predicted with both assessment methods, i.e. the models for the observed errors and the *k*-NN method. The standwise RMSEs and biases were first predicted with both assessment methods. After that RMSEs and biases of the three test data sets were obtained by adding together the standwise estimates of the RMSEs and biases, and by dividing the sums by the number of the stands. The predictions of uncertainty were compared to empirical RMSEs and biases.

The considered stand characteristics were D_{gM}, H_{gM}, BA and V. The predictions and related uncertainty measures were produced for stands without logging and silvicultural treatments and together for the stands with and without treatments, using both methods, i.e. the models for the observed errors and the *k*-NN method.

The uncertainty assessments obtained by both methods, i.e. the models for the observed errors method and the *k*-NN method, were also studied at the stand level. The 95% confidence intervals of the updated stand characteristics were estimated for each stand based on the uncertainty assessment of the stand characteristics depending on the method used. After estimating the confidence intervals, the proportions of the observed errors included in the estimated confidence intervals were studied by using both methods. The studied updating period was 10 years.

3 Results

3.1 Models for Observed Errors

The models for the observed errors (Eqs. 2–3) were estimated in the first phase for the true stand-level inventory data (INKA1). Here the modelling was carried out by first excluding the stands in which management operations had been performed during the updating time (Table 6). Then the modelling was carried out in whole INKA1 data (Table 7). The bias models (Eq. 5) for the growth predictions of the stand characteristics were also estimated for both cases (Tables 8,9). The uncertainty of the growth predictions increased with the applied updating period. In addition, the relative uncertainty of the growth predictions was clearly higher for young stands. The effect of treatments was considerable: growth and yield predictions were more difficult in treated stands. The uncertainty of the updated BA and V were assessed more reliable than the uncertainty of the updated median tree characteristics.

In the second phase, the models of the observed errors were estimated using stand-level inventory data CONTROL1, which included generated measurement errors. Now the treatment effect was not as clear as it has been when using true stand-level inventory data (Tables 10,11). Moreover, the effect of the updating time on the uncertainty of the stand characteristics had diminished. Furthermore, the bias models for the growth predictions of the stand characteristics were estimated for data with and without the stands in which management operations had been performed during the updating time (Tables 12,13).

Table 6. The models of the observed errors of the growth predictions of the stand characteristics with the correct stand inventory data (INKA1) without treated stands.

Model	R^2	R^2_{adj}	Std. error	Predictors	Coefficients	Std. error of coeff.
Ln(V $error^2$)	0.30	0.30	2.664	Constant	–0.305 *	0.211
				T_i (a)	0.381 ***	0.015
				BA	0.05507 ***	0.016
				V(i)	0.01215 ***	0.001
				BA_{PI}	–0.738 ***	0.155
				Age	0.004529 ***	0.002
				dd	–0.0003519 ***	0.000
				V	0.0004854 ***	0.002
				SC_4	0.262 ***	0.113
BA $error^2$	0.16	0.16	5.265	Constant	1.733 ***	0.426
				T_i (a)	0.436 ***	0.029
				Age	–0.007329 *	0.004
				BA_{BI}	3.009 ***	0.554
				BA(i)	0.165 ***	0.028
				Species	0.289 **	0.105
				D_{gM}	–0.148 ***	0.032
				$SC_{1,2}$	0.916 **	0.356
				V	0.1081 ***	0.004
				BA	–0.06588 **	0.033
D_{gM} $error^2$	0.07	0.07	2.453	Constant	2.077 ***	0.381
				T_i (a)	0.092 ***	0.016
				BA_{PI}	–1.917 ***	0.356
				BA_{SP}	–1.483 ***	0.383
				$(D_{gM})^2$	0.0032 ***	0.000
				V	–0.01096 ***	0.002
				SC_4	–0.355 **	0.132
				BA	0.04205 **	0.020
ln(H_{gM} $error^2$)	0.10	0.10	2.455	Constant	–1.312 **	0.610
				T_i (a)	0.139 ***	0.016
				Species	0.174 **	0.085
				BA_{PI}	–0.316 *	0.172
				BA_{MAX}	–1.845 ***	0.504
				D_{gM}	0.03718 ***	0.010
				H_{gM}(i)	0.09542 ***	0.032
				BA_{BI}	0.756 **	0.347

T_i(a), Growth prediction time, years; BA, Basal area, $m^2 ha^{-1}$; V(i), Estimated growth of mean stand volume, $m^3 ha^{-1}$; BA_{PI}, Proportion of Scots pine in stand basal area, (0–1); Age, Stand age, years; dd, Total annual temperature sum; V, Mean stand volume, $m^3 ha^{-1}$; SC_x, Fertility class according to Kuusela and Salminen (1969), dummy-variable in which x = 1,2 ... 8. Definition of x values: 1 = Very rich, 2 = Rich, 3 = Moist, 4 = Dryish, 5 = Dry, 6 = Barren, 7 = Rocky sites, sandy sites and alluvial soils, 8 = Hilltops and fells; H_{gM}, Height of basal area median tree, m; BA_{BI}, Proportion of birch in stand basal area, (0–1); BA(i), Estimated growth of stand basal area, $m^2 ha^{-1}$; Species, Number of tree species in stand; D_{gM}, Basal area median diameter, cm; BA_{SP}, Proportion of Norway spruce in stand basal area, (0–1); BA_{MAX}, Maximum proportion of basal area of tree species in stand basal area, (0–1); H_{gM}(i), Estimated height growth of basal area median tree, m; *, coefficient is significant at 0.05 level; **, coefficient is significant at 0.01 level, ***, coefficient is significant at 0.001 level

Table 7. The models of the observed errors of the growth predictions of the stand characteristics with the correct stand inventory data (INKA1) with treated stands.

Model	R^2	R^2_{adj}	Std. error	Predictors	Coefficients	Std. error of coeff.
ln(V error^2)	0.38	0.38	2.638	Constant	–0.878 **	0.392
				T_i (a)	0.382 ***	0.014
				BA	0.0335 *	0.018
				Treat	2.29 ***	0.159
				V(i)	0.009143 ***	0.001
				BA_{PI}	–0.778 ***	0.134
				dd	–0.0003805	0.000
				V	0.004856 ***	0.001
				ln(V)	0.318 **	0.126
				$SC_{1,2}$	–0.341 **	0.161
ln(BA error^2)	0.69	0.69	3.255	Constant	20.536 ***	2.541
				T_i (a)	1.174 ***	0.019
				East	–0.000003532 ***	0.000
				dd	–0.003604 ***	0.000
				Treat	2.392 ***	0.192
				BA_{BR}	3.837 **	1.899
				BA	0.05744 **	0.024
				H_{gM}	–0.112 ***	0.021
				$SC_{1,2}$	0.629 **	0.252
				SC_3	0.277 *	0.157
				SC_{4+}	–0.620 **	0.252
				ln(BA)	–0.649 **	0.284
D_{gM} error^2	0.11	0.11	3.283	Constant	0.725 **	0.326
				Treat	0.790 ***	0.229
				BA_{PI}	–1.121 ***	0.209
				T_i (a)	0.114 ***	0.022
				$(D_{gM})^2$	0.004078 ***	0.000
				V	–0.009106 ***	0.001
				BA(i)	–0.146 ***	0.022
				D_{gM}(i)	0.230 ***	0.037
				Species	0.239 ***	0.080
ln(H_{gM} error^2)	0.10	0.10	2.426	Constant	–1.407 **	0.533
				T_i (a)	0.150 ***	0.015
				BA_{MAX}	–1.963 ***	0.454
				BA	0.03275 ***	0.006
				H_{gM}(i)	0.09878 ***	0.028
				BA_{BI}	0.678 **	0.284
				Species	0.177 **	0.075

Treat, dummy-variable in which 0 = Stand not treated during growth prediction time, 1 = Stand treated during growth prediction time; East, Y coordinate, m; BA_{BR}, Proportion of broadleaves in stand basal area, (0–1); D_{gM}(i), Estimated growth of basal area median diameter, cm;

Table 8. The bias models of the growth predictions of the stand characteristics with the correct stand inventory data (INKA1) without treated stands.

Model	R^2	R^2_{adj}	Std. error	Predictors	Coefficients	Std. error of coeff.
V	0.20	0.19	13.46	Constant	–2.225 *	2.713
				Age	–0.194 ***	0.017
				T_i (a)	0.934 ***	0.160
				SC_3	4.062 **	0.835
				BA	2.522 ***	0.267
				V(i)	–0.431 ***	0.000
				V^2	0.00246 ***	0.155
				D_{gM}	1.529 ***	0.159
				V	–0.349 ***	0.041
				Ln(BA)	–6.877 ***	1.870
				Species	–1.865 ***	0.449
				SC_5	–4.681 ***	1.347
BA	0.17	0.16	1.578	Constant	0.921 ***	0.223
				Age	–0.2435 ***	0.002
				BA(i)	–0.125 ***	0.014
				SC_5	–1.459 ***	0.169
				T_i (a)	0.154 ***	0.021
				SC_4	–0.530 **	0.099
				$(D_{gM})^2$	0.009718 **	0.001
				D_{gM}	0.981 **	0.017
				V	–0.09889 ***	0.002
				dd	–0.003055 **	0.000
D_{gM}	0.13	0.13	1.030	Constant	1.149 ***	0.135
				Age	–0.06807 ***	0.001
				V	0.09646 ***	0.001
				Species	–0.153 ***	0.034
				SC_5	–0.433 ***	0.103
				BA	–0.4641 ***	0.010
				dd	–0.001376 **	0.000
				SC_3	–0.135 **	0.063
				D_{gM}	–0.4524 **	0.100
H_{gM}	0.14	0.13	1.302	Constant	1.267 ***	0.378
				$(H_{gM})^2$	–0.1025 ***	0.001
				D_{gM}	0.184 ***	0.017
				Age	–0.06955 ***	0.002
				V	0.1461 ***	0.003
				$\ln(H_{gM})$	–1.172 ***	0.257
				H_{gM}(i)	–0.5798 ***	0.017
				Ti (a)	0.3899 **	0.015
				BA_{BI}	–0.5223 **	0.022

V(i), Estimated growth of stand mean volume, $m^3 ha^{-1}$

Table 9. The bias models of the observed errors of the growth predictions of the stand characteristics with the correct stand inventory data (INKA1) with treated stands.

Model	R^2	R^2_{adj}	Std. error	Predictors	Coefficients	Std. error of coeff.
V	0.36	0.36	19.8	Constant	6.241 **	3.727
				Treat	15.795 ***	1.315
				V(i)	–0.179 ***	0.013
				Age	–0.306 ***	0.023
				BA_{SP}	8.542 ***	1.799
				D_{gM}	1.351 ***	0.190
				T_i (a)	1.502 ***	0.225
				SC_5	–10.701 ***	2.119
				ln(BA)	–7.813 ***	2.542
				BA	2.070 ***	0.332
				V	–0.180 ***	0.025
				SC_4	–4.721 ***	1.313
				Species	–1.775 ***	0.578
				dd	–0.02747 **	0.001
BA	0.39	0.39	2.287	Constant	3.132 ***	0.339
				BA(i)	–0.304 ***	0.015
				Age	–0.3598 ***	0.003
				BA_{PI}	–0.694 ***	0.217
				T_i (a)	0.188 ***	0.026
				SC_5	–2.413 ***	0.342
				Treat	0.890 ***	0.160
				SC_4	–1.078 ***	0.273
				dd	–0.003734 ***	0.000
				D_{gM}	0.7275 ***	0.019
				V	–0.04655 ***	0.001
				BA_{BR}	–0.921 ***	0.320
				SC_3	–0.441**	0.218
D_{gM}	0.13	0.12	1.175	Constant	1.278 ***	0.155
				Age	–0.08555 ***	0.001
				D_{gM} (i)	–0.808 ***	0.012
				V	0.07698 ***	0.001
				Species	0.004078 ***	0.033
				D_{gM}	–0.5687 ***	0.010
				SC_5	–0.480 ***	0.106
				BA	–0.3242 ***	0.010
				BA_{BR}	2.126 **	0.842
				Treat	–0.187 ***	0.071
				T_i (a)	0.289 **	0.012
H_{gM}	0.13	0.12	1.316	Constant	1.329 ***	0.328
				$(H_{gM})2$	–0.07747 ***	0.001
				V	0.06993 ***	0.001
				D_{gM}	0.147 ***	0.015
				$Ln(H_{gM})$	–1.107 ***	0.192
				Age	–0.0587 ***	0.002
				$H_{gM}(i)$	–0.8539 ***	0.017
				Ti (a)	0.6044 ***	0.013
				SC_1	0.307 **	0.113
				Species	–0.108 ***	0.036
				SC_5	–0.28 **	0.118

Table 10. The models of the observed errors of the growth predictions of the stand characteristics with the erroneous stand inventory data (CONTROL1) without treated stands.

Model	R^2	R^2_{adj}	Std. error	Predictors	Coefficients	Std. error of coeff.
ln(V error^2)	0.16	0.15	2.413	Constant	–3.024 ***	0.434
				T_i (a)	0.133 ***	0.015
				Alt	0.002387 ***	0.001
				dd	0.003198 ***	0.000
				BA_{SP}	0.190 ***	0.094
				V_{level}	0.00995 ***	0.004
				SC_{5+}	–0.312 ***	0.102
				Age	0.003948 ***	0.001
				ln(V)	0.762 ***	0.049
				1/V	1.539 ***	0.509
				$SC_{1,2}$	0.463 ***	0.108
				V(i)	–0.005416 ***	0.002
BA error^2	0.04	0.04	2.874	Constant	–2.721 ***	0.243
				T_i (a)	0.09566 ***	0.011
				BA	0.874 ***	0.084
				BA(i)	0.08338 ***	0.014
				BA_{SP}	–0.803 ***	0.090
				Species	0.114 ***	0.035
				dd	0.000671 ***	0.000
				D_{gM}	–0.023 ***	0.006
				$1/D_{gM}$	0.800 ***	0.228
				BA_{BR}	1.895 **	0.951
D_{gM} error^2	0.07	0.07	2.453	Constant	2.077 ***	0.381
				T_i (a)	0.092 ***	0.016
				BA_{PI}	–1.917 ***	0.356
				BA_{SP}	–1.483 ***	0.383
				$(D_{gM})^2$	0.0032 ***	0.000
				V	–0.01096 ***	0.002
				SC_4	–0.355 **	0.132
				BA	0.04205 *	0.020
ln(H_{gM} error^2)	0.10	0.10	2.455	Constant	–1.312 **	0.610
				T_i (a)	0.139 ***	0.016
				Species	0.174 **	0.085
				BA_{PI}	–0.316 **	0.172
				BA_{MAX}	–1.845 ***	0.504
				D_{gM}	0.03718 ***	0.010
				H_{gM}(i)	0.09542 ***	0.032
				BA_{BI}	0.756 **	0.347

Alt, Elevation above sea level, m; V_{level}, Expected RMSE of mean stand volume of measurer;

Table 11. The models of the observed errors of the growth predictions of the stand characteristics with the erroneous stand inventory data (CONTROL1) with treated stands.

Model	R^2	R^2_{adj}	Std. error	Predictors	Coefficients	Std. error of coeff.
Ln(V $error^2$)	0.20	0.19	2.375	Constant	–2.05 ***	0.532
				T_i (a)	0.0903 ***	0.010
				Alt	0.001946 ***	0.001
				dd	0.002916 ***	0.000
				BA_{SP}	0.329 ***	0.082
				V_{level}	0.006558 *	0.004
				SC_{5+}	–0.289 ***	0.092
				Age	0.003771 ***	0.001
				1/V	1.303 **	0.628
				$SC_{1,2}$	0.456 ***	0.088
				Treat	0.753 ***	0.062
				V^2	–0.0000167 ***	0.000
				V	0.007001 **	0.003
				ln(V)	0.557 ***	0.121
LN(BA $error^2$)	0.12	0.11	2.310	Constant	–3.010 ***	0.437
				Treat	1.170 ***	0.062
				BA	–0.205 ***	0.046
				BA (i)	0.08406 ***	0.008
				D_{gM}	–0.031 ***	0.006
				dd	0.001195 ***	0.000
				ln(BA)	1.964 ***	0.315
				$(BA)^2$	0.004286 ***	0.001
				1/BA	1.545 ***	0.317
				BA_{MAX}	–0.413 ***	0.174
				BA_{SP}	–0.216 ***	0.081
				BA_{BI}	1.347 ***	0.684
Var(D_{gM} $error^2$)	0.11	0.11	3.283	Constant	0.725 **	0.326
				Treat	0.790 ***	0.229
				BA_{PI}	–1.121 ***	0.209
				T_i (a)	0.114 ***	0.022
				$(D_{gM})^2$	0.004078 ***	0.000
				V	–0.009106 ***	0.001
				BA(i)	–0.146 ***	0.022
				D_{gM}(i)	0.230 ***	0.037
				Species	0.239 ***	0.080
ln(H_{gM} $error^2$)	0.10	0.10	2.426	Constant	–1.407 **	0.533
				T_i (a)	0.150 ***	0.015
				BA_{MAX}	–1.963 ***	0.454
				BA	0.03275 ***	0.006
				H_{gM}(i)	0.09878 ***	0.028
				BA_{BI}	0.678 **	0.284
				Species	0.177 **	0.075

Table 12. The bias models of the growth predictions of the stand characteristics with the erroneous stand inventory data (CONTROL1) without treated stands.

Model	R^2	R^2_{adj}	Std. error	Predictors	Coefficients	Std. error of coeff.
V	0.22	0.22	29.615	Constant	–121.896 ***	5.668
				BA_{SP}	30.643 ***	2.511
				V^2	–0.004594 ***	0.000
				SC_1	40.183 ***	1.770
				V (a)	–0.541 ***	0.021
				dd	0.8476 ***	0.004
				T_i (a)	2.601 ***	0.177
				SC_3	14.372 ***	1.023
				BA_{PI}	23.766 ***	2.533
				Alt	0.6643 ***	0.008
				D_{gM}	1.759 ***	0.126
				H_{gM}	–1.892 ***	0.173
BA	0.29	0.29	3.117	Constant	–2.227 ***	0.559
				BA(a)	–0.836 ***	0.028
				BA_{PI}	–0.926 ***	0.147
				BA	0.239***	0.000
				T_i (a)	0.286 ***	0.019
				SC_5	–4.62 ***	0.223
				SC_4	–2.917 ***	0.179
				SC_3	–1.415 ***	0.144
				Alt	0.08066 ***	0.001
				BA_{BI}	–2.585 ***	0.267
				V(a)	–0.3517	0.004
D_{gM}	0.24	0.24	1.926	Constant	–4.74 ***	0.374
				D_{gM}	–0.325 ***	0.009
				H_{gM}	0.204 ***	0.010
				SC_1	1.455 ***	0.089
				D_{gM}(i)	–0.568 ***	0.032
				BA_{BI}	–1.168 ***	0.169
				dd	0.05001 ***	0.000
				Alt	0.07384 ***	0.001
				Age	0.1185 ***	0.001
				T_i (a)	0.115 ***	0.012
				BA_{MAX}	1.196 ***	0.164
H_{gM}	0.40	0.40	1.766	Constant	–2.438 ***	0.357
				$(H_{gM})2$	–0.158 ***	0.000
				D_{gM}	0.284 ***	0.007
				dd	0.05714***	0.000
				H_{gM}(i)	–0.793 ***	0.023
				$Ln(H_{gM})$	–2.986 ***	0.108
				V	0.1339 ***	0.001
				T_i (a)	0.161 ***	0.010
				SC_1	0.995 ***	0.077
				Alt	0.05511 ***	0.000

Table 13. The bias models of the growth predictions of the stand characteristics with the erroneous stand inventory data (CONTROL1) with treated stands.

Model	R^2	R^2_{adj}	Std. error	Predictors	Coefficients	Std. error of coeff.
V	0.33	0.33	30.941	Constant	–121.047 ***	4.477
				V(i)	–0.631 ***	0.012
				SC_1	24.554 ***	1.129
				dd	0.104 ***	0.003
				V^2	–0.004403 ***	0.000
				BA_{SP}	20.921 ***	1.043
				T^i (a)	2.856 ***	0.139
				Alt	0.8775 ***	0.007
				D_{gM}	2.056 ***	0.114
				H_{gM}	–2.124 ***	0.155
BA	0.39	0.39	3.233	Constant	–3.951 ***	0.522
				BA (i)	–0.598 ***	0.009
				BA_{PI}	–0.732 ***	0.130
				Ln(BA)	–0.9984 *	0.129
				dd	0.09298 ***	0.000
				T_i (a)	0.253 ***	0.014
				SC_5	–4.464 ***	0.197
				SC_4	–2.838 ***	0.155
				BA	–0.198 ***	0.012
				Alt	0.09313 ***	0.001
				SC_3	–1.454 ***	0.121
D_{gM}	0.28	0.28	1.939	Constant	–4.171 ***	0.297
				D_{gM}	–0.338 ***	0.008
				H_{gM}	0.198 ***	0.009
				SC_1	0.114 ***	0.022
				$(D_{gM})2$	1.489 ***	0.072
				T_i (a)	0.171 ***	0.010
				dd	0.05584 ***	0.000
				Alt	0.08039 ***	0.000
				BA_{BI}	–1.528 ***	0.129
				Age	0.1069 ***	0.001
H_{gM}	0.42	0.42	1.776	Constant	–3.986 ***	0.322
				$(H_{gM})2$	–0.1632 ***	0.000
				D_{gM}	0.241 ***	0.007
				dd	0.06845 ***	0.000
				H_{gM}(i)	–0.801 ***	0.020
				$Ln(H_{gM})$	–2.823 ***	0.097
				V	0.1324 ***	0.000
				T_i (a)	0.185 ***	0.008
				SC_1	1.213 ***	0.064
				Alt	0.06629 ***	0.000
				Age	0.09503 ***	0.001

3.2 Assessment of the Uncertainty of the Updated Inventory Data

The uncertainty in the prediction error of the updated stand characteristics was assessed by using the true stand-level inventory data INKA1. Both methods, i.e. the models of observed errors and the *k*-NN method, where applied. When the updating time was 5 years, both methods produced RMSEs close to the observed RMSEs (Table 14). The growth and yield of BA and V of the treated stands were considerably underestimated. Both methods reflected this in their bias predictions. When the updating time was extended to 10 years, the *k*-NN method gave RMSEs for the updated stand characteristics that were clearly closer to the observed RMSEs of the treated and untreated stands when compared to the models of the observed errors method (Table 15). The models of the observed errors tended to overestimate the RMSEs of the basal areas and mean volumes, and to underestimate the RMSEs of the mean diameters and mean heights. The bias predictions were in line with the observed biases in both methods.

The assessment of the uncertainty of the updated stand characteristics was also carried out with two test data sets. In the case of the CONTROL2 data, the models of the observed errors produced slight underestimates of the pre-

Table 14. The observed and predicted RMSEs and biases of the updated stand characteristics of the true stand level inventory data (INKA1) after a growth prediction period of 5 years.

	RMSE Observed	Models	*k*-NN method	Bias Observed	Models	*k*-NN method
No treatment						
D_{gM} (cm)	0.6 (3.7)	1.2 (7.5)	0.6 (3.4)	–0.05 (–0.3)	–0.2 (–1.1)	–0.08 (–0.5)
H_{gM} (m)	1.1 (8.6)	0.8 (5.9)	1.1 (8.3)	0.1 (0.7)	–0.3 (–2.1)	0.1 (0.9)
BA ($m^2 ha^{-1}$)	1.4 (7.6)	1.8 (9.5)	1.2 (6.5)	–0.1 (–0.8)	–0.1 (–0.6)	–0.2 (–1.0)
V ($m^3 ha^{-1}$)	10.2 (8.0)	9.8 (7.6)	8.2 (6.4)	0.01 (0.01)	–1.1 (–0.8)	–0.1 (0.1)
Treatment						
D_{gM} (cm)	1.6 (9.6)	1.6 (9.1)	1.2 (6.9)	–0.2 (–1.1)	–0.2 (–0.9)	–0.1 (–0.8)
H_{gM} (m)	1.3 (9.2)	0.8 (5.6)	1.2 (8.8)	0.1 (0.8)	–0.2 (–1.1)	0.2 (1.3)
BA ($m^2 ha^{-1}$)	5.1 (25.7)	2.6 (15.0)	5.0 (25.0)	2.5 (12.5)	2.4 (13.8)	3.1 (15.8)
V ($m^3 ha^{-1}$)	41.0 (27.6)	27.3 (21.6)	38.4 (25.9)	21.9 (14.8)	22.5 (17.8)	27.8 (18.7)

Table 15. The observed and predicted RMSEs and biases of the updated stand characteristics of the true stand level inventory data after a growth prediction period of 10 years.

	RMSE Observed	Models	*k*-NN method	Bias Observed	Models	*k*-NN method
No treatment						
D_{gM} (cm)	1.0 (5.5)	1.5 (8.3)	0.9 (5.2)	–0.1 (–0.7)	–0.2 (–1.1)	–0.2 (–1.0)
H_{gM} (m)	1.4 (10.4)	1.0 (7.3)	1.4 (10.4)	0.3 (2.2)	–0.3 (–2.2)	0.3 (2.3)
BA ($m^2 ha^{-1}$)	2.4 (11.6)	2.4 (11.5)	2.1 (10.4)	–0.2 (–0.9)	0.3 (1.4)	–0.2 (–1.1)
V ($m^3 ha^{-1}$)	17.0 (11.6)	25.5 (17.5)	16.2 (11.0)	1.2 (0.8)	2.2 (1.5)	1.0 (0.7)
Treatment						
D_{gM} (cm)	2.5 (13.4)	1.8 (9.9)	1.7 (9.2)	0.2 (1.0)	–0.1 (–0.6)	0.1 (0.5)
H_{gM} (m)	1.8 (12.2)	1.1 (7.2)	1.7 (11.1)	0.5 (3.3)	–0.1 (–0.8)	0.5 (3.3)
BA ($m^2 ha^{-1}$)	5.2 (25.4)	8.0 (44.0)	5.1 (25.0)	2.1 (10.4)	3.0 (16.2)	2.6 (12.7)
V ($m^3 ha^{-1}$)	43.7 (27.4)	46.7 (34.4)	42.6 (26.7)	23.1 (14.5)	27.2 (20.1)	27.0 (16.9)

Table 16. The observed and predicted RMSEs and biases of the updated stand characteristics of the test data (CONTROL2) with errors of the stand-level inventory after a growth prediction period of 5 years.

	RMSE Observed	Models	*k*-NN method	Bias Observed	Models	*k*-NN method
No treatment						
D_{gM} (cm)	2.1 (12.9)	1.6 (10.0)	2.0 (12.3)	0.2 (1.1)	0.6 (3.8)	0.3 (2.0)
H_{gM} (m)	2.1 (16.6)	1.7 (13.0)	2.1 (16.7)	0.2 (1.2)	0.2 (1.3)	0.1 (0.7)
BA ($m^2 ha^{-1}$)	3.3 (18.1)	2.8 (15.4)	3.3 (18.6)	0.3 (1.8)	1.7 (9.1)	0.1 (0.6)
V ($m^3 ha^{-1}$)	28.8 (22.6)	27.9 (21.8)	28.0 (22.7)	2.3 (1.2)	3.6 (2.8)	–0.05 (–0.4)
Treatment						
D_{gM} (cm)	2.3 (14.2)	2.0 (12.2)	2.2 (12.9)	0.5 (3.0)	0.8 (4.9)	0.6 (3.4)
H_{gM} (m)	2.4 (17.2)	1.3 (9.7)	2.3 (16.3)	0.3 (2.0)	0.3 (2.5)	0.3 (2.1)
BA ($m^2 ha^{-1}$)	5.0 (25.6)	4.7 (24.4)	5.0 (23.4)	1.9 (9.9)	2.7 (13.9)	2.2 (10.4)
V ($m^3 ha^{-1}$)	45.3 (32.6)	33.6 (24.2)	42.8 (27.5)	16.7 (12.0)	13.2 (9.5)	19.3 (12.4)

Table 17. The observed and predicted RMSEs and biases of the updated stand characteristics of the test data (CONTROL2) with errors of the stand level inventory after a growth prediction period of 10 years.

	RMSE Observed	Models	*k*-NN method	Bias Observed	Models	*k*-NN method
No treatment						
D_{gM} (cm)	2.3 (13.3)	1.7 (9.9)	2.4 (13.4)	0.2 (1.0)	0.5 (2.6)	0.3 (1.9)
H_{gM} (m)	2.3 (17.0)	1.8 (13.2)	2.3 (16.8)	0.4 (3.1)	0.3 (2.4)	0.4 (3.2)
BA ($m^2 ha^{-1}$)	4.0 (20.2)	3.4 (17.4)	3.9 (19.2)	0.7 (3.3)	1.8 (9.1)	0.5 (2.4)
V ($m^3 ha^{-1}$)	33.9 (24.0)	34.8 (24.7)	33.3 (22.6)	6.2 (4.4)	5.2 (3.7)	4.6 (3.1)
Treatment						
D_{gM} (cm)	2.8 (16.1)	2.3 (13.2)	2.8 (15.1)	0.9 (5.2)	0.7 (3.9)	0.9 (4.9)
H_{gM} (m)	2.7 (18.8)	1.5 (10.2)	2.6 (17.1)	0.6 (4.5)	0.4 (2.7)	0.8 (5.3)
BA ($m^2 ha^{-1}$)	5.7 (31.8)	5.8 (32.5)	5.6 (25.9)	3.0 (16.6)	4.6 (25.5)	3.4 (15.6)
V ($m^3 ha^{-1}$)	53.7 (40.3)	43.7 (32.8)	54.1 (31.9)	29.4 (22.1)	27.5 (20.6)	33.8 (19.9)

Table 18. The observed and predicted RMSEs and biases of the updated stand characteristics of the test data (NORTH2) with errors of the stand level inventory from the NORTH data after a growth prediction period of 5 years.

	RMSE Observed	Models	*k*-NN method	Bias Observed	Models	*k*-NN method
No treatment						
D_{gM} (cm)	1.9 (11.5)	1.6 (9.4)	2.0 (12.3)	–0.2 (–1.0)	0.4 (2.3)	0.3 (1.8)
H_{gM} (m)	2.3 (18.4)	1.8 (14.2)	2.2 (17.2)	0.1 (0.5)	0.1 (0.8)	0.1 (0.6)
BA ($m^2 ha^{-1}$)	2.8 (16.3)	2.9 (16.9)	3.1 (17.7)	0.7 (4.1)	1.9 (11.1)	0.3 (1.6)
V ($m^3 ha^{-1}$)	27.2 (22.7)	27.2 (22.7)	27.8 (22.6)	3.2 (2.7)	3.8 (3.1)	0.1 (0.1)
Treatment						
D_{gM} (cm)	2.6 (14.9)	2.0 (11.7)	2.2 (12.6)	–0.2 (–0.9)	0.5 (2.8)	0.5 (2.7)
H_{gM} (m)	2.6 (18.7)	1.5 (10.9)	2.3 (16.1)	0.3 (1.9)	0.4 (2.9)	0.3 (1.6)
BA ($m^2 ha^{-1}$)	4.6 (24.2)	4.8 (25.1)	5.0 (23.6)	2.2 (11.5)	2.8 (14.9)	2.5 (11.7)
V ($m^3 ha^{-1}$)	43.9 (32.1)	34.2 (25.0)	44.0 (28.3)	18.7 (13.7)	15.3 (11.2)	20.4 (13.1)

Table 19. The observed and predicted RMSEs and biases of the updated stand characteristics of the test data (NORTH2) with errors of the stand level inventory from the NORTH data after a growth prediction period of 10 years.

	RMSE Observed	Models	*k*-NN method	Bias Observed	Models	*k*-NN method
No treatment						
D_{gM} (cm)	2.2 (12.4)	1.7 (9.7)	2.3 (12.9)	–0.2 (–1.1)	0.3 (1.6)	0.2 (1.2)
H_{gM} (m)	2.5 (18.7)	1.9 (14.2)	2.3 (16.8)	0.3 (2.1)	0.2 (1.4)	0.4 (2.6)
BA ($m^2 ha^{-1}$)	3.6 (18.6)	3.4 (17.9)	3.9 (18.9)	1.2 (6.3)	2.1 (10.7)	0.7 (3.5)
V ($m^3 ha^{-1}$)	33.1 (23.9)	35.6 (25.7)	34.1 (23.1)	8.6 (6.2)	6.1 (4.4)	5.0 (3.4)
Treatment						
D_{gM} (cm)	3.3 (18.0)	2.5 (13.1)	2.7 (14.6)	0.01 (0.05)	0.2 (0.9)	0.6 (3.2)
H_{gM} (m)	2.9 (18.8)	1.7 (11.2)	2.6 (17.0)	0.5 (3.3)	0.4 (2.6)	0.7 (4.6)
BA ($m^2 ha^{-1}$)	5.6 (27.0)	5.9 (33.0)	5.7 (26.5)	3.1 (14.7)	4.8 (26.7)	3.8 (17.4)
V ($m^3 ha^{-1}$)	57.2 (35.3)	45.8 (33.9)	55.2 (32.4)	30.4 (18.7)	30.9 (22.8)	35.9 (21.0)

dicted RMSEs of the stand characteristics when the updating time was 5 years (Table 16), whereas the predicted RMSEs were close to the observed RMSEs when using the k-NN method in both the treated and untreated stands. The prediction was somewhat more difficult to perform when dealing with the treated stands. When the updating time was extended to 10 years, the accuracy of the k-NN method was still quite good (Table 17). When the updating time was increased, the relative RMSEs increased slightly, but the relative biases increased significantly. The biases of the updated stand characteristics could be predicted using both methods.

The uncertainty of the updated stand characteristics was also estimated using the NORTH2 stand-level inventory data. With an updating time of 5 years, the predictions of RMSEs of V and BA produced by both methods were quite similar to the observed RMSEs in the untreated stands (Table 18). The predicted RMSEs of D_{gM} and H_{gM} were slight underestimates. The predicted biases were in the same direction as the observed biases for BA and V. When the updating time was extended to 10 years, the predicted RMSEs of D_{gM} and H_{gM} became underestimates (Table 19). The predicted RMSE of V of the untreated stands was overestimated when using the model of observed errors. The predicted biases of the stand characteristics were quite similar to the observed biases.

The uncertainty assessments of the updated stand characteristics were also made at stand level. Confidence intervals of 95% were derived from the assessments of uncertainty for every stand. The proportions of stands in which the prediction of uncertainty of $D_{gM,}$ BA and V in the correct stand inventory data (INKA1) when using the models of observed errors were within the confidence intervals were greater when compared to the proportions achieved by using the *k*-NN method (Table 20). Contrary to this, the proportions of stands in which the prediction of uncertainty of H_{gM} were within the confidence intervals were greater when using the *k*-NN method. Similar differences were found when assessing the first stand inventory data including measurement errors (CONTROL2), although the proportions of stands, including these derived confidence intervals, were somewhat smaller (Table 20). When the measurement errors were generated from the NORTH data, the proportions of the stands within the confidence intervals were a slightly greater when using the model of observed errors compared to the CONTROL2 data (Table 20). On the other hand, when the same comparison was made with the *k*-NN method, the proportions of the stands were smaller. The confidence intervals were considerably more unfavourable with treated stands.

Table 20. The percentage of the stands within the 95% confidence interval derived from the uncertainty assessments with the INKA1 data, CONTROL1 data and NORTH2 data.

		D_{gM}	H_{gM}	BA	V
INKA1	Modelling observed errors, no treatments, 5 years	99.5	88.3	99.1	95.8
	k-NN method, no treatments, 5 years	93.2	90.1	91.6	90.5
	Modelling observed errors, treatments, 5 years	98.2	86.7	82.3	89.4
	k-NN method, treatments, 5 years	90.3	92.9	71.7	70.8
	Modelling observed errors, no treatments, 10 years	98.7	83.7	97.8	96.9
	k-NN method, no treatments, 10 years	91.9	90.2	89.3	89.3
	Modelling observed errors, treatments, 10 years	96.1	81.2	99.5	97.6
	k-NN method, treatments, 10 years	88.2	89.2	84.8	84.8
CONTROL2	Modelling observed errors, no treatments, 5 years	94.0	92.4	93.8	95.8
	k-NN method, no treatments, 5 years	92.0	92.7	92.3	92.7
	Modelling observed errors, treatments, 5 years	98.2	86.7	89.4	89.4
	k-NN method, treatments, 5 years	89.3	93.0	79.4	86.8
	Modelling observed errors, no treatments, 10 years	93.7	91.9	92.8	95.1
	k-NN method, no treatments, 10 years	92.4	94.0	90.6	90.2
	Modelling observed errors, treatments, 10 years	93.0	83.6	98.0	94.5
	k-NN method, treatments, 10 years	86.4	89.0	79.7	84.3
NORTH2	Modelling observed errors, no treatments, 5 years	96.5	96.7	94.5	96.7
	k-NN method, no treatments, 5 years	93.4	94.2	92.5	94.0
	Modelling observed errors, treatments, 5 years	97.5	89.3	99.2	95.1
	k-NN method, treatments, 5 years	89.3	90.9	83.5	88.1
	Modelling observed errors, no treatments, 10 years	96.4	96.2	93.7	96.2
	k-NN method, no treatments, 10 years	91.1	92.4	89.7	89.3
	Modelling observed errors, treatments, 10 years	98.7	90.2	98.3	93.2
	k-NN method, treatments, 10 years	85.9	90.6	78.6	82.5

4 Discussion

This paper is a study of two methods for assessing the uncertainty of updated forest inventory data with the forest simulator. The studied methods were (i) the models for observed errors and (ii) the *k*-NN method. The uncertainty assessments of growth and yield predictions using both methods were found to be feasible when dealing with large stand data sets. Furthermore, the confidence intervals for the uncertainty predictions of the updated stand characteristics for individual stands were satisfactory for both methods.

When comparing the above results to results reported in previous literature, Kangas (1999) found that the models for observed errors were also feasible for assessing the uncertainty of growth and yield predictions. Furthermore, Haara (2002) achieved promising results for the uncertainty assessments of growth predictions by compounding the *k*-NN method and multiobjective optimisation. However, these two uncertainty prediction methods have not been compared previously. In addition, the study data used by Kangas (1999) were confined to even-aged pure pine stands.

The increased need for valid growing stock data is leading to continuous updating of stand databases. In continuous updating, the stand attributes are estimated in the field following a forestry operation (cutting or silvicultural treatment) and then they are stored in databases (Koivuniemi and Korhonen 2006). It is inevitable that the continuous growth simulation of stands increases the uncertainty of growing stock data when compared to actual field measurements. Predictions of the uncertainty of the updated stand data are useful as decision support when discussing planning problems and the quality of planning data.

The INKA data applied in this study have been used as modelling data in many growth models in connection with forestry simulators (Hynynen et al. 2002, Redsven et al. 2004). However, this study focused on the stand-level approach and the growth models used were individual-tree-based growth models. The values of the dependent variables were assumed to be error-free. This assumption could be made since measurement errors in the dependent variables do not introduce bias into the coefficients, but only slightly increase the variance of the models (Carroll et al. 1995).

The main advantage of both of the studied methods is that the bias and the accuracy of the predictions can be assessed. However, both methods require independent and repeatedly measured inventory data. This can be seen as being the main drawback of the studied methods when comparing them to methods such as the Monte Carlo simulation or Taylor series approximations (e.g. Gertner 1987, Gertner and Dzialowy 1984, Mowrer, 1991, Kangas 1997, 1998). On the other hand, both of the tested methods are quite easy to adapt to forest simulation systems if only contemporary models and distance functions are estimated. Furthermore, using these methods does not considerably add to the computation time even when doing growth predictions for large areas.

Forest simulators, as well as the models included in the simulators, are usually under continuous development and the accuracy and the precision of the models can change over time. This implies that the empirical estimates related to the observed errors produced in this study should be also updated to correspond to the current situation. Furthermore, the models of observed errors should be updated using new data. On the other hand, the use of the *k*-NN method is very flexible in dealing with changing situations. If the assumptions related to the distance function are valid, one needs to update only the reference data with simulators using current calculation practices.

In addition to the procedures studied in this study for deriving inventory data, it would be also possible to utilise old inventory data with automatic interpretation of remote sensing material such as medium resolution satellite images and high resolution aerial photographs (e.g. Anttila 2002, Hyvönen 2002). Old inventory data can provide additional information from stands (e.g. fertility class, regeneration age, biodiversity), which can be compound especially with obtained laser scanner data, whose accuracy of the stand characteristics is adequate for planning purposes (e.g. Maltamo et al. 2004, Næsset 2004). Numerical interpretation of remote sensing material helps in avoiding the subjectivity of stand-level inventory and in reducing inventory costs. However, choosing the timing of forest management operations is still an unresolved problem when using image interpretation techniques only. This is a problem especially in the Nordic countries where forest stands are managed under regulated management schedules with several thinnings included.

This study focused on errors in planning data, but the next step could be an approach where the errors are connected to the costs and benefits of the planning situation. An example of such an approach is the cost-plus-loss analysis (e.g. Ståhl et al. 1994, Eid 2000, Holmström et al. 2003, Eid et al. 2004) in which the total costs include the costs of the inventories and the expected loss of non-optimal decisions based on uncertain forest data. Thus, whenever the additional costs of inventory and planning are less than the additional income, it would be useful to carry out an additional inventory. In addition to the errors in basic forestry data and the uncertainty in predicting forest development, the sources of uncertainty in forest planning calculations include the uncertainty of the future prices paid for timber (e.g. Leskinen and Kangas 1998) and the uncertainties involved in forest owners' preferences (e.g. Leskinen 2001). Separate analyses are important, but there is also a need for aggregate consideration of all these sources of uncertainty, to explore the possible interactions of the different uncertainty sources, and to find those aspects of uncertainty, which are the most critical in decision support. This, in turn, facilitates adaptive planning with respect to critical uncertainty sources. For example, the high uncertainty of stand attributes predicted by either one of the methods considered in this study could lead to adaptive planning with new measurements of the stand attributes of some stands.

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Total of 61 references