Biomass Equations for Birch in Finland

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Biomass equations were compiled for the above- and below-ground tree components of birch (*Betula pendula* Roth and *Betula pubescens* Ehrh.). The equations were based on 127 sample trees in 24 birch stands located on mineral soil sites. The study material consisted of 20 temporary plots and ten plots from four thinning experiments with different thinning intensities (unthinned, moderately and heavily thinned plots).

The equations were estimated for the following individual tree components: stem wood, stem bark, living and dead branches, foliage, stump, and roots. In the data analysis, a multivariate procedure was applied in order to take into account the statistical dependency among the equations. Three multivariate variance component models were constructed for the above-ground biomass components, and one for the below-ground biomass components. The multivariate model (1) was mainly based on tree diameter and height, and in the multivariate models (2) and (3) additional commonly measured tree variables were used.

The equations provided logical biomass predictions for a number of tree components, and were comparable with other functions used in Finland and Sweden. The applied statistical method generated equations that gave more reliable biomass estimates than the equations presented earlier. Furthermore, the structure of the multivariate models enables more flexible application of the equations, especially for research purposes.

1 Introduction

Interest in estimating tree and forest biomass for practical forestry (e.g., energy resources) and for calculating forest carbon budgets at the national or international scale, as well as for research purposes (e.g. the estimation of nutrient cycling and fluxes, the carbon cycle), has increased during the last few decades. Because direct measurement of the tree biomass or its components (usually expressed as dry weight of stem, crown, stump and roots), is time-consuming and expensive, allometric regression functions for tree biomass or its components have been developed as a function of easily measurable tree variables.

Several studies on birch biomass have been published in the Nordic countries. However, only a few of the functions are based on representative material and include all the main biomass components. In Finland, there has been a lack of the widely applicable (general) individual-tree biomass models, especially models for stump and roots and for the foliage of birch (Kärkkäinen 2005). In Sweden, Marklund (1988) published biomass functions for different above-ground components of birch, excluding foliage, on the basis of a large material from the Swedish national forest inventory. These functions are commonly used in Scandinavia and, according to Kärkkäinen (2005), they are also applicable in Finland. In Finland, Hakkila's (1979, 1991) functions have also often been applied for predicting crown and stem biomass. Hakkila's (1991) functions for crown biomass are primarily applicable to trees in logging removals. Hakkila's (1979) functions for stem biomass are based on a large, representative material gathered as a part of the 5th Finnish National Forest Inventory (1969–1970). Repola et al. (2007) published birch biomass functions for above- and below-ground components based on a material collected over a large part of Finland. Functions for the above-ground biomass components of birch, based on a more limited material (regional, a narrow range of diameter of sample trees, for single biomass components only), have been published by Mälkönen (1977), Simola (1977), Björklund and Ferm (1982), Mälkönen and Saarsalmi (1982), Finer (1989), Laiho (1997), Starr et al. (1998) in Finland, and by Johansson (1999), Petersson (1999), Cleasson et al. (2001), and Petersson (2006) in Sweden. Functions for the below-ground biomass components of birch have been published only by Petersson (2006) and Repola et al. (2007).

Biomass models should meet specific requirements before they can be used in forest management planning systems and forest biomass inventories at the national scale (Kärkkäinen 2005). First, the models have to be widely applicable in giving reliable biomass estimates of the total tree and the tree components: stem wood, stem bark, living and dead branches, foliage, stump, and roots. Second, the biomass models should be based on the variables that are normally measured in forest inventories, or which can be estimated easily and reliably from inventory data. Third, the models should be based on the same sample trees in order to give reliable estimates of the individual biomass components. Applying models based on separate sample trees can distort the relationships between the tree components.

In model estimation, information about the study material should be utilized efficiently in order to obtain reliable estimates of the parameters. Biomass data are usually hierarchically structured and are based on sample trees collected in different stands. Tree properties (biomass components) usually vary from stand to stand, and are more strongly correlated within stands than between stands. The fact that the data have a hierarchical structure has frequently been ignored, and the models have been fitted using the ordinary least squares (OLS) method resulting in too optimistic a view of the reliability of the parameter estimates. A more precise procedure is to apply the generalized least squares (GLS) estimation method, which usually yields more precise estimates of the models parameters, and permits analysis of the between-stand and within-stand variation with the random stand and tree effects (Lappi 1991).

In general the models for the biomass of tree components are fitted separately, i.e. independently. This model approach is based on the assumption of statistical independency among the biomass components in the same stand or tree. Because this assumption is not valid, a better procedure is to take into account the contemporaneous correlation (correlation of errors in the different equations) and estimate the parameters of the equations simultaneously. By applying

a multivariate procedure (linear or non-linear seemingly unrelated regression) more reliable estimates can be produced for fixed parameters (Zellner 1962, Parresol 1999 and 2001, Carvalho et al. 2003, Bi et al. 2004, Návar et al. 2004). In addition, the fixed prediction can be calibrated to a new observation by utilizing across-equation covariance and a measurement of the other dependent variable (Lappi 1991). A desirable feature in the equations of tree components is that the sum of the predictions for the tree component equals the prediction for the whole tree (Parresol 1999). Biomass additivity can be ensured in a multivariate model by setting across-equation constraints (i.e. linear restrictions on the regression coefficients) (Briggs 1984, Parresol 1999, Carvalho et al. 2003, Bi et al. 2004, Návar et. al. 2004).

The aim of this study is to compile individual-tree biomass equations for the above- and below-ground tree components of birch by applying a multivariate procedure. Another aim is also to study whether the multivariate procedure gives more reliable parameter estimates than the separately (independently) estimated equations published by Repola et al. (2007).

2 Material

2.1 Study Material

The study material consisted of 24 birch stands: 20 temporary plots and four thinning experiments, representing a large part of Finland (Fig. 1). The stands were mainly located on mineral soil and represented sites that ranged from moderately to highly productive in terms of site quality. Three stands were on peatlands and one stand on earlier cultivated land. The stands in the thinning experiments had been planted and most of the temporary plots were naturally regenerated. The stands were mainly dominated by *Betula pendula* or *Betula pubecens*, with a variable admixture of Norway spruce or Scots pine (Table 1). The stands were even-aged, and ranged from pole age stands to mature stands (Table 1).

Temporary plots were selected in five of the Finnish Forest Research Institute's research areas, located in different parts of Finland. Four
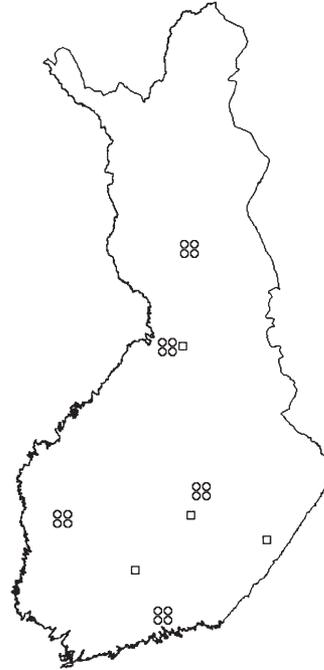

Fig. 1. The location of the study stands. Temporary plots are marked with a circle and thinning experiments with a square.

Table 1. Range of stand characteristics.

Variable	Mean	Std. dev.	Min	Max
T, year	47	22.7	11	97
G, $m^2 ha^{-1}$	19.2	6.4	2.7	32.3
D, cm	17.7	5.8	4.2	30.2
H, m	17.2	5.7	4.8	25.9
H_{dom}, m	18.6	6.0	5.5	28.7
Birch, %	75.0	24.2	28.1	100.0

T = age at stump height, G = stand basal area, D = mean diameter at breast height (weighted with tree basal area), H = mean height (weighted with tree basal area), H_{dom} = height of dominant trees, birch = proportion of birch out of basal area (%).

temporary plots, selected in young to mature stands, were established in each research area. Pure birch stands, mixed pine and birch stands or mixed spruce and birch stands were accepted. The temporary plots were located subjectively in representative parts of the stands. All the sample plots were circular plots with a 7-meter radius in a young stand and a 15-meter radius in more advanced stands.

The thinning experiments, which were located in pure birch stands, consisted of plots with different thinning intensities. Unthinned and heavily thinned plots were selected in each experiment, and moderately thinned plots in two of the experiments. Trees growing in the buffer zone of the plots were selected as sample trees.

2.2 Sample Trees

The total number of sample trees was 127; 85 trees from the temporary plots and 42 from the thinning experiments. The majority of the sample trees were silver birch (66%) and a minor part were pubescent birch (34%). The sample trees, in most cases 4–5 trees per plot, represented the whole growing stock, but were selected randomly by weighting by tree size, i.e. trees were selected with a probability proportional to d^2 (d = breast height diameter). Damaged trees were not accepted as sample trees. The diameter and age distribution of the sample trees was broad, the diameter ranging between 2.5 and 38.7 cm (Table 2).

The fieldwork was carried out 2002 and 2003. Tree age, height, living crown length, stem diameter and bark thickness at six points along the stem, and breast height diameter increment during the last five years (i_5) were measured on each sample tree. Sample disks were taken at breast height and at a height of 70% for stem biomass determination.

The living crown was divided into four sections of equal length, and one living sample branch was selected subjectively from each section to represent the average-sized (diameter and height) branch of the crown section. One dead sample branch per tree was taken from the lowest crown section. All the remaining branches in the crown section were cut off and divided into living and dead branches. The fresh weight of the branches in each section was measured in the field. The sample branches were taken to the laboratory for fresh and dry weight determination. After 2–3 days drying at a temperature of 70 °C, the dry mass of branches was determined by weighing.

The stump and root biomasses were measured on a sub-sample of the trees on the temporary plots. Stump biomass included both the above-

Table 2. Sample trees characteristics.

Variable	Mean	Std	Range
Diameter, cm	16.5	7.0	2.5–38.0
Height, m	17.1	6.2	3.9–29.0
Age [a]	37	20.6	7–132
Crown ratio, (0–1)	0.58	0.14	0.29–0.96
Radial growth [b], cm	0.75	0.58	0.05–3.47
Bark thickness [c], cm	0.9	0.48	0.2–2.8

[a] Age measured at breast height
[b] Breast height radial increment during the last five years
[c] Double bark thickness at breast height

and belowground components, and the average stump height was 16 cm and 1% of the tree height. The minimum coarse root diameter varied from 2–5 cm depending on the tree diameter. In addition, the root biomass was determined on roots with a diameter larger than 1 cm on six trees. The fresh weight of the stump and roots were determined in the field. For moisture content determination one sample was taken from the stump (sector) and two discs from the roots.

3 Methods

3.1 Biomass Estimation on the Sample Trees

The biomass was estimated by individual tree components; stem wood, stem bark, living and dead branches, foliage, stump and roots. The branch biomass included both branch wood and bark. Not all the biomass components were measured on all the sample trees (Table 3).

Crown

The branch biomass of the tree was estimated by applying a stratified ratio estimator (Cochran 1977, Parresol 1999). The ratio of the dry and fresh weight of the sample branches was used to estimate separately the branch and foliage dry weight from the fresh mass. Ratio estimates for living branch biomass were first calculated by crown sections. The total living branch biomass was the sum of the individual crown sections. A constant moisture content, based on the mean

Table 3. The number of sample trees used in measuring the biomass components.

	Number of sample trees
Stem wood	127
Stem bark	127
Living branch	127
Dead branch	127
Foliage	21
Stump	39
Roots: >2–5 cm	39
>1 cm	6

moisture content of dead sample branches on the plots, was used for dead branches.

Stem Wood

Stem wood biomass was calculated by multiplying the stem volume by the average stem wood density. Stem volume, both under-bark and over-bark, was calculated by utilizing Laasasenaho's (1982) taper curve equations calibrated with measured dimensions of the sample trees (height and diameter at six points along the stem).

Wood density (kg m^{-3}) of the sample tree was measured on two sample disks taken at breast height and at a height of 70%. Owing to the risk of bias in the estimates of average wood density, which was determined on the basis of only two sample disks per tree, the average wood density was determined by utilizing, in addition to the basic density measurement of two sample disks, the equation for the vertical dependence of wood density (Repola 2006) and the stem taper curve. Repola's (2006) equations were calibrated with the measurements made on the two disks in order to obtain a tree level density curve, which depicted the wood density at different points along the stem. The corresponding stem diameters, which were used as a weight in estimating the average wood density, were obtained from the taper curve. The average wood density was then calculated from the density curve and taper curve.

Stem Bark

The biomass of stem bark was obtained from the average bark density and bark volume of the tree.

The bark volume of the stem was calculated as the difference between the under-bark and over-bark stem volume. Bark volume was based on the measured bark dimensions of the sample discs. The mean bark density of the sample tree was the mean of the bark density measurements made on the two sample disks (breast height and a height of 70%). Disk level bark density was obtained by dividing the bark dry mass by the bark volume.

Stump and Roots

The stump and root biomass was measured on 39 sample trees. The coarse root biomass, i.e. roots >2–5 cm, was measured on all 39 sample trees, and the biomass of roots >1 cm was determined on six sample trees, the breast height diameter of which ranged from 5 to 25 cm. The stump and root biomasses of the tree were estimated by applying a ratio estimation method based on the moisture content of the samples and the measured fresh weight of the roots and stump. First a simple regression equation (1) was constructed for the dependence of roots >1 cm on the coarse roots (2–5 cm) on the basis of six sample trees. Then the >1 cm root biomass was estimated for the whole root material by applying the compiled equation (1).

$$y = 1.068 + 1.364\,x \qquad R^2 = 0.99, \ \hat{\sigma} = 1.698 \text{ kg} \qquad (1)$$

where y is the >1 cm root biomass and x is the biomass of coarse roots >2–5 cm.

3.2 Model Approach

The basic assumption in our model approach was that biomass components on the same site and in the same tree are dependent. This meant statistical dependency among the equations, i.e. the errors of the individual equations are correlated. Multi-variate procedures with random parameters were applied to take into account the across-equation correlation and to obtain more reliable parameter estimates compared to equations estimated independently (Parresol 1999).

Equations for stem wood, stem bark, foliage, living and dead branches and total above-ground tree biomass were compiled. Equations for below-

ground biomass components were estimated for stump, roots (> 1 cm) and total below-ground biomass. First the equations for biomass components and total above-ground and below-ground biomass were fitted independently (single models). Then a set of linear models was constructed to form a multivariate linear model. The parameters of the multivariate models were estimated simultaneously. The compiled multivariate model was written as follows:

$$
\begin{aligned}
y_{ki1} &= \mathbf{b}_1 \mathbf{x}_{1ki} + u_{1k} + e_{1ki} \\
y_{2ki} &= \mathbf{b}_2 \mathbf{x}_{2ki} + u_{2k} + e_{2ki} \\
&\;\;\vdots \\
y_{nki} &= \mathbf{b}_n \mathbf{x}_{nki} + u_{nk} + e_{nki}
\end{aligned}
\tag{2}
$$

where

$y_{1ki},\, y_{2ki}\dots y_{nki}$	= dependent variables of biomass component 1, 2, ... n for tree i in stand k
n	= number of biomass components
$\mathbf{x}_{1ki},\, \mathbf{x}_{2ki}\dots \mathbf{x}_{nki}$	= vectors of the independent variables for tree i in stand k
$\mathbf{b}_1,\, \mathbf{b}_2\dots \mathbf{b}_n$	= vectors of the fixed effects parameters
$u_{1k},\, u_{2k}\dots u_{nk}$	= random effects for stand k
$e_{1ki},\, e_{2ki}\dots e_{nki}$	= random effects for tree i in stand k (residual error)

The covariance components, $\mathrm{cov}(u_{jk},\, u_{j+1k})$ and $\mathrm{cov}(e_{jki},\, e_{j+1ki})$, which illustrated the dependency among the random effects of biomass components j, were estimated for both the stand and tree level. All the random parameters ($u_{1k},\, u_{2k}\dots u_{nk}$) of the same stand are correlated with each other, and the residuals errors ($e_{1ki},\, e_{2ki}\dots e_{nki}$) of the same tree are correlated. The random parameters and residuals errors are assumed to be uncorrelated, and also assumed to be identically distributed Gaussian random variables with a mean of 0. In addition, the random parameters are assumed to have difference variances. The model assumptions were checked by visual inspection.

The material was hierarchically, 2-level (temporary plots) and 3-level (thinning experiments), structured. To define the model we treated the study stand as a level 2 unit (between site) and the tree (within site) as a 1 level unit. In order to simplify the structure of the data the plots in

the thinning experiments were assumed to be independent. MIXED procedures of SAS (SAS Institute 1999) were used to estimate the multivariate models.

First the equations for tree components, total above-ground and below-ground biomass were estimated separately and independently. Then a set of linear models (i.e. the multivariate linear model) was constructed, the parameters of which were estimated simultaneously. To analyse the efficiency of the multivariate procedure in model estimation, the standard error due to the uncertainty of the parameter estimates ($\mathrm{SE}_{\mathrm{parametric}}$), and indicating the prediction reliability, was calculated for the equations estimated independently (single equations) and using the multivariate procedure. $\mathrm{SE}_{\mathrm{parametric}}$ was calculated for each observation as follows:

$$
\mathrm{SE}_{\mathrm{parametric}} = \sqrt{\mathbf{x}_{ki}\, \mathrm{var}(\hat{\mathbf{b}})\mathbf{x}_{ki}}
\tag{3}
$$

where, $\mathbf{x}_{ki}$ is a vector of the independent variables of tree i at site k, and $\mathrm{var}(\hat{\mathbf{b}}) = (\mathbf{X}^T \mathbf{V}^{-1} \mathbf{X})^{-1}$ is the covariance matrix of the fixed-effects parameter estimates, $\mathbf{X}$ is the matrix of the fixed regressors, and $\mathbf{V}$ is the variance-covariance matrix for the random effects including both site and tree effects.

The biomass equations have a multiplicative model form. Logarithmic transformation was used to obtain homoscedasticity of the variance, and to transform the equation to a linear form. When applying the fixed part of the equations, a variance correction term, $\left(\sigma_u^2 + \sigma_e^2\right)/2$ (where $\sigma_u^2 = \mathrm{var}(u_k)$ and $\sigma_e^2 = \mathrm{var}(e_{ki})$), should be added to the intercept in order to correct for the bias due to the logarithmic transformation. This correction factor tended to lead to overestimation of the biomass of dead branches due to the large variance value, $\left(\sigma_u^2 + \sigma_e^2\right)$. An empirical correction term (c) was calculated from the data using the formula $c = \sum y / \sum e^{\ln(\hat{y})}$, where y is the measured biomass of the dead branches and $\hat{y}$ is the fixed prediction for dead branches. The prediction can then be retransformed to the linear scale with the correction term as follows: $y = e^{\ln(\hat{y})} \cdot c$ (Baskerville 1972).

3.3 Model Application

The model for biomass component 1 in one stand can be expressed in matrix form as follows:

$$\mathbf{y}_1 = \mathbf{ì}_1 + \mathbf{Z}_1 \mathbf{u}_1 + \mathbf{e}_1 \qquad (4)$$

where $\mathbf{y}_1$ is a vector of the observed values of a dependent variable (a biomass component), $\mathbf{ì}_1$ is a fixed mean vector for a biomass component, $\mathbf{u}_1$ is the vector of the random effects for a biomass component with $E(\mathbf{u}_1) = 0$, and var$(\mathbf{u}_1) = \mathbf{D}$, $\mathbf{e}_1$ is the vector of random errors with $E(\mathbf{e}_1) = 0$ and var$(\mathbf{e}_1) = \mathbf{R}$, and $\mathbf{Z}_1$ is the model matrix of the random variables. If the measurements for a biomass component $\mathbf{y}_1$ were observed then the vector of the random effects $\mathbf{u}_1$ can be predicted on the basis of linear prediction theory (Henderson 1953, McCulloch and Searle 2001, Lappi 1991):

$$\mathbf{u}_1 = (\mathbf{Z}^T \mathbf{R}^{-1} \mathbf{Z} + \mathbf{D}^{-1})^{-1} \mathbf{Z}^T \mathbf{R}^{-1} (\mathbf{y}_1 - \mathbf{ì}_1) \qquad (5)$$

If a random measurement error exists in $\mathbf{u}_1$, then it can be taken into account by adding its variance to the diagonal of $\mathbf{R}$ (Lappi 1986). When the vector of the random effects of a biomass component ($\mathbf{u}_1$) and its measurement error variance vector var$(\mathbf{u}_1)$ are predicted, the fixed predictions of another biomass component j ($\hat{\mathbf{ì}}_j$) can be calibrated by predicting the vector of the random effects $\mathbf{u}_j$. The vector of the random effects $\mathbf{u}_j$ is then predicted by utilizing the across-equation covariance cov$(\mathbf{u}_1,\mathbf{u}_j)$. Assume that $E(\mathbf{u}_j) = \mathbf{ì}_{uj}$ with $E(\mathbf{u}_j) = 0$, $\mathbf{T} = \text{cov}(\mathbf{u}_1,\mathbf{u}_j)$, and $\mathbf{D} = \text{var}(\mathbf{u}_1)$. Then the best linear unbiased predictor (BLUP) of $\mathbf{u}_j$ is (Lappi 1991):

$$\mathbf{u}_j = \mathbf{ì}_{uj} + \mathbf{T}\mathbf{D}^{-1}\mathbf{u}_1 \qquad (6)$$

4 Results

4.1 Multivariate Models

Multivariate models were constructed separately for the above-ground and below-ground biomass. Owing to the different number of observations of the above- and below-ground components, the model parameters could not be estimated simultaneously. The multivariate model for above-ground

biomass contained the equations for stem wood, stem bark, living and dead branches and total tree biomass. As the equation for foliage biomass was estimated independently due to the limited material, foliage biomass was not included in the total above-ground biomass. The multivariate model for below-ground biomass included stump, roots with diameter > 1 cm and total below-ground biomass (stump and roots).

Three multivariate variance component models for above-ground biomass and one for below-ground biomass were constructed. Multivariate model (1) was based only on tree diameter at breast height (d) and tree height (h). Multivariate model (2) contained, in addition to diameter and height, tree age at breast height (t_{13}) and crown variables, crown length (cl) or crown ratio (cr) as independent variables (see Appendix). Multivariate model (3) was based, in addition to the previously mentioned variables, on bark thickness (bt) and radial increment during the last five years (i_5) (see Appendix).

In model formulation the most significant independent variable, diameter at breast height, was expressed as an approximation of the stump diameter, $d_S = 2 + 1.25d$ (Laasasenaho 1982), which can be interpreted as a transformation rather than an estimate of stump diameter. This was done in order to obtain a model that is also valid for trees with a height under 1.3 m. The best transformation of stump diameter was $d_S/(d_S + m)$, where m is a constant determined by the grid search method. Marklund (1988) used the same transformation based on breast height diameter. A similar transformation, in addition to ln(h), also proved to be a usable expression of tree height .

Multivariate Model 1a

Above-ground biomass equations (Table 4):

Stem wood:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 12)} + b_2 \ln(h_{ki}) + u_{1k} + e_{1ki} \quad (7)$$

Stem bark:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 12)} + b_2 \frac{h_{ki}}{(h_{ki} + 20)} \qquad (8)$$
$$+ u_{2k} + e_{2ki}$$

Living branches:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_k}{(d_k + 16)} + b_2 \frac{h}{(h + 10)} \qquad (9)$$
$$+ u_{3k} + e_{3ki}$$

Dead branches:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_k}{(d_k + 16)} + u_{4k} + e_{4ki} \qquad (10)$$

Total aboveground:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 12)} + b_2 \frac{h_{ki}}{(h_{ki} + 22)} \qquad (11)$$
$$+ u_{5k} + e_{5ki}$$

Separate model 1

Foliage:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 2)} + u_{6k} + e_{6ki} \qquad (12)$$

Multivariate Model 1b

Below-ground biomass equations (Table 5):

Stump:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 26)} + u_{7k} + e_{7ki} \qquad (13)$$

Roots > 1 cm:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 22)} + b_2 \ln(h_{ki}) \qquad (14)$$
$$+ u_{8k} + e_{8ki}$$

Total belowground:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 24)} + b_2 \ln(h_{ki}) \qquad (15)$$
$$+ u_{9k} + e_{9ki}$$

where
y_{ki} = biomass component or total biomass of tree i in stand k, kg
d_{ki} = tree diameter at breast height of tree i in stand k, cm
$d_{Ski} = 2 + 1.25\ d$, cm
h_{ki} = tree height of tree i in stand k, m

4.2 Model Evaluation

In all the equations the between-stand variance was clearly lower than the within-stand variance (Tables 4, 5, A1 and A2). The equations were not directly comparable because multivariate model (3) was based on fewer observations, due to missing measurements of independent variables, compared to multivariate model (1) and (2). In general, the addition of independent variables to the equation reduced the between-stands variance more than the within-stand variance.

The addition of independent variables to Eq. (7) for stem biomass based on diameter and height reduced only the between-stands variance and not the within-stand variance. Adding the interaction between tree diameter and age (depicting the tree growth rate) to the stem wood biomass equation (A1) decreased the between-stand variance and total error variance by 33% and 13%, respectively. The equation for stem biomass had a similar form to that of multivariate models (2) and (3). Using bark thickness at breast height (*bt*) as the independent variable improved the bark equation (A8) by decreasing the total error variance by 15%.

Eqs. (9) and (12) for living branches and foliage were improved significantly by adding crown variables to Eqs. (A3) and (A6). Using the crown ratio (*cr*) as an independent variable reduced the total error variance in the foliage equation (A6) by 29%. Similarly, the addition of crown length (*cl*) to the branch equation (A3) decreased the between- and within-stand variance by 44% and 26%, respectively. The total error variance decreased by about 24% more when the radial increment (i_5) and tree age (t_{13}) were added to the living branch equation (A9). The total error variance was considerable in the equations for dead branches, and Eq. (10) based on tree diameter improved by only 16% when tree age and radial increment were added to Eq. (A10).

4.3 Correlation among the Equations

The equations for biomass components were not independent, i.e. covariance (correlation) was detected between the random parameters at both the stand and tree levels (residual error). In general, the across-equation correlation at the stand

Table 4. The parameter estimates of multivariate model 1a. For the fixed parameters the standard error is given in parentheses. Variances and covariances of random stand parameters (u_{nk}) and residual errors (e_{nki}), and the empirical correction factor (c) for dead branches models, are given.

	Stem wood	Stem bark	Living branches	Dead branches	Total above-ground	Foliage
	Eq. (7)	Eq. (8)	Eq. (9)	Eq. (10)	Eq. (11)	Eq. (12)
Fixed	N=127	N=127	N=127	N=127	N=127	N=21
b_0	–4.879	–5.401	–4.152	–8.335	–3.654	–29.566
	(0.065)	(0.150)	(0.220)	(1.141)	(0.053)	(3.881)
b_1	9.651	10.061	15.874	12.402	10.582	33.372
	(0.162)	(0.460)	(0.580)	(1.966)	(0.146)	(4.201)
b_2	1.012	2.657	–4.407	-	3.018	-
	(0.042)	(0.504)	(0.642)		(0.150)	
Random	u_{1k}	u_{2k}	u_{3k}	u_{4k}	u_{5k}	u_{6k}
u_{1k}	0.00263					
u_{2k}	0.00001	0.01043				
u_{3k}	–0.00328	0.00732	0.02733			
u_{4k}	–0.03529	–0.04965	–0.04136	1.11490		
u_{5k}	0.00099	0.00104	0.00184	–0.02976	0.00068	
u_{6k}						0.000
	e_{1ki}	e_{2ki}	e_{3ki}	e_{4ki}	e_{5ki}	e_{6ki}
e_{1ki}	0.00544					
e_{2ki}	0.00434	0.04443				
e_{3ki}	0.00612	–0.00089	0.07662			
e_{4ki}	0.00573	–0.01302	–0.0523	2.6789		
e_{5ki}	0.00532	0.00900	0.01346	0.00607	0.00727	
e_{6ki}						0.077
c				2.0737		

Table 5. The parameter estimates of multivariate model 1b. For the fixed parameters the standard error is given in parentheses. Variances and covariances of random stand parameters (u_{nk}) and residual errors (e_{nki}) are given.

	Stump	Roots >1 cm	Stump and roots >1 cm
	Eq. (13)	Eq. (14)	Eq. (15)
Fixed	N=127	N=127	N=127
b_0	–3.574	–3.223	–2.726
	(0.233)	(0.472)	(0.374)
b_1	11.304	6.497	7.652
	(0.528)	(0.853)	(0.707)
b_2	-	1.033	0.799
		(0.273)	(0.217)
Random	u_{7k}	u_{8k}	u_{9k}
u_{7k}	0.02154		
u_{8k}	–0.0163	0.0480	
u_{9k}	–0.00742	0.03469	0.02623
	e_{7ki}	e_{8ki}	e_{9ki}
e_{7ki}	0.04542		
e_{8ki}	0.009156	0.02677	
e_{9ki}	0.0166	0.02283	0.02152

level was higher than that at the tree level. The highest correlations between the above-ground biomass components occurred between the stem wood and living as well as dead branches, for which the random parameters at the stand level had uniform negative correlation in all multivariate models: they varied from –0.387 to –0.752. This means that, in the stands where stem wood biomass was overestimated, the branch biomass tended to be underestimated (Fig. 2). Corresponding negative correlation (–0.507) was also detected between the stump and roots. The tree level errors were only slightly correlated among the biomass components, and no correlations over 0.500 were detected. Random parameters of the total tree biomass were highly correlated with the other biomass components, especially with stem wood biomass, at both the stand and tree levels.
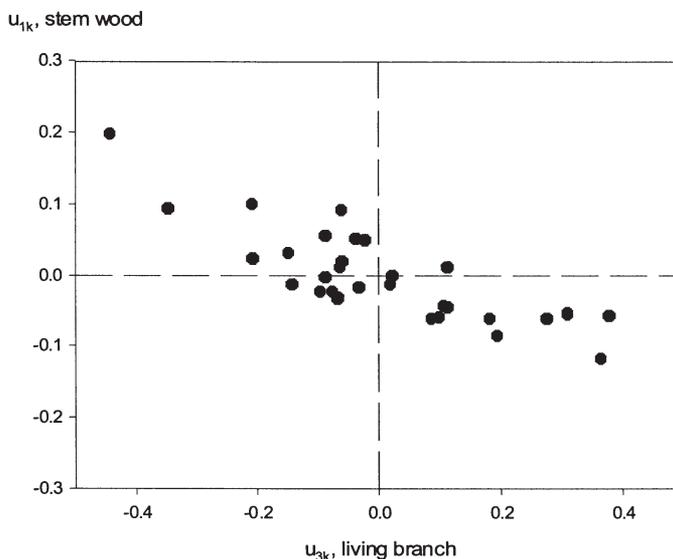
Fig. 2. The correlations (–0.752) among stand level random parameters of the equations for stem wood and living branch biomasses in multivariate model (3).

Table 6. The average standard error due to the uncertainty of the parameter estimates ($SE_{parametric}$) on a logarithmic scale for different tree components when the equations were estimated independently (single models 1, 2 and 3) or by applying the multivariate procedure (multivariate models 1, 2 and 3).

	Multivariate model 1	Single models 1	Multivariate models 2	Single models 2	Multivariate model 3	Single models 3
Stem	0.0163	0.0171	0.0153	0.0169	0.0165	0.0169
Bark	0.0407	0.0420	0.0409	0.0420	0.0418	0.0415
Living branches	0.0578	0.0631	0.0494	0.0519	0.0508	0.0546
Dead branches	0.3072	0.3040	0.3045	0.3040	0.3892	0.3981
Total above	0.0133	0.0147	0.0135	0.0148	0.0135	0.0153
Stump	0.0745	0.0744				
Roots	0.1021	0.1032				
Stump and roots	0.0792	0.0807				

4.4 Multivariate vs. Single Models

As a consequence of the across-equation correlation, the multivariate models overall produced more reliable parameter estimates compared to the independently (single models) estimated equations. This also means a slightly lower standard error ($SE_{parametric}$) due to the smaller uncertainty of the parameter estimates, especially for living branches, produced by the multivariate models compared with that produced by the single models

(Table 6). In any case, the multivariate models reduced the relative standard error ($SE_{parametric}$) of the predictions, depending on the tree component, by at the most by 0.9 percentage unit and, on the average, by only 0.2 percentage unit.

4.5 Biomass Additivity

A desirable feature of tree components equations is that the sum of predictions for the tree
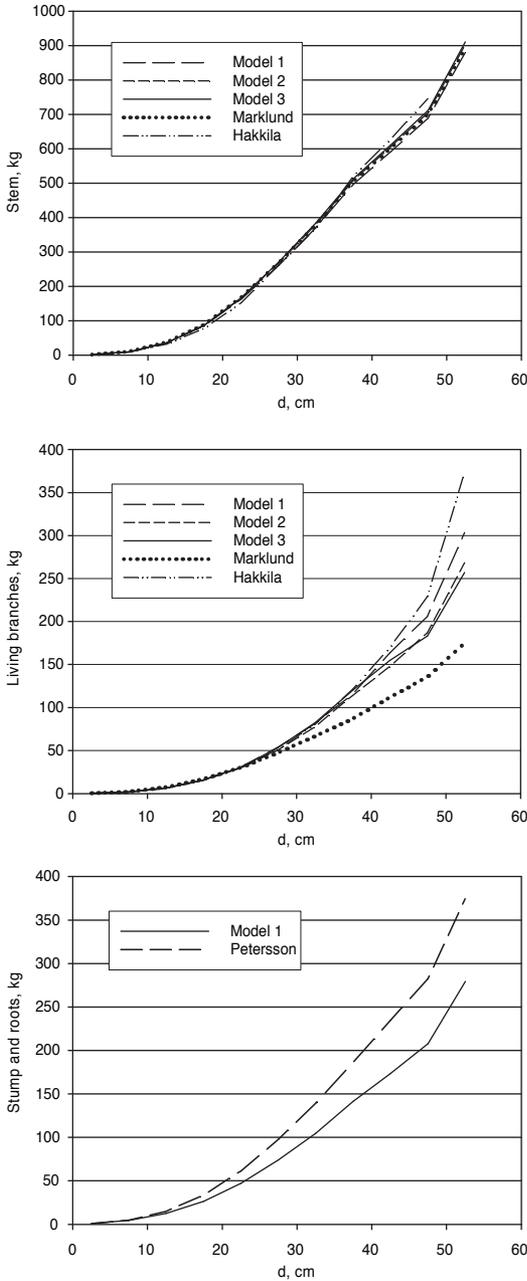
Fig. 3. The expected tree biomass in the Finnish NFI data predicted using different biomass functions.

components equals the prediction for the whole tree (Parresol 1999). The sum of predictions for the tree components equalled the prediction for the total tree biomass relatively well without parameter restrictions, even though there was variation from stand to stand. When applying multivariate models (1) and (2), the sum of the tree components resulted in an average of 0.2% and 0.3% lower tree biomass compared to the prediction for the total tree equations (11) and (A5). In multivariate model (3) the tree component equations produced on average of 0.2% higher tree biomass compared to the prediction for the total tree equation (A11).

4.6 Comparison with Other Functions

The predictions given by the equations compiled in this study were compared with the results obtained with other functions commonly used in Finland (Hakkila 1979, 1991, Marklund 1988, Petersson 1999, 2006). The reference functions for the above-ground tree components were based on tree diameter and height, and the functions for the below-ground tree components on tree diameter. Sample trees from the 9th Finnish National Forest Inventory (NFI9, 1996–2003) data were used as the test material.

For stem biomass, including wood and bark, all the functions gave relatively similar results, although Hakkila's (1979) function gave the highest stem biomass for trees >40 cm (Fig. 3). The comparisons were made for functions in which only the variation in stem form, caused by varying breast height diameter and height, was taken into account.

For living branches Marklund's (1988) functions gave the lowest and Hakkila's (1991) functions the highest biomass (Fig. 3). When foliage was included in the crown biomass, multivariate models (2) and (3) and Petersson's (2006) functions predicted similar crown biomass over the diameter range, and multivariate model (1) gave the highest crown biomass for trees with a diameter >40 cm.

The equations compiled for below-ground biomass gave, on the average, 30% lower stump and root biomass compared with of the values with Petersson's (2006) function (Fig. 3). In our study

Table 7. Tree characteristics of the sample trees used in the model demonstration.

Plot	Tree	d, cm	d_6, cm	h, m	t_{13}	$v_{(d,h,d6)}$	$v_{(d,h)}$
4_11	22	12.2	8.3	14.2	46	72.1	79.3
4_11	31	17.5	14.3	17.2	49	194.8	193.3
4_11	43	15.8	12.4	16.1	47	146.1	147.6
4_11	46	19.0	16.1	18.2	47	246.4	238.5
Average		16.1	12.8	16.4	47.3	164.9	164.7
357_11	5	23.8	17.8	16.5	25.0	311.3	321.3
357_11	6	18.8	15.1	18.6	24.0	230.3	239.7
357_11	7	20.1	15.7	18.1	26.0	249.8	263.9
Average		20.9	16.2	17.7	25.0	263.8	275.0

root biomass was determined up to a diameter of 1 cm, and in Petersson's (2006) study up to a diameter of 5 mm.

4.7 Model Application – Calibration of the Equations with Random Parameters

The applicability of the multivariate model was demonstrated by predicting random stand parameters for two study stands, where tree diameter (d), diameter at a height of six meter (d_6), height (h) and tree age (t_{13}) were measured (Table 7). The biomass of the tree components can be calculated using multivariate model (1) based on tree diameter and height. In addition, d_6 and t_{13} can also be utilized for calculating the stem wood biomass (SW) as a product of stem volume and wood density. Stem volume ($v_{(d,d6,h)}$) was calculated as a function of d, d_6 and h (Laasasenaho 1982), and the average stem wood density (without bark) as a function of d and t_{13} (Repola et. al 2007). Variance of the logarithmic SW is the sum of the variances of the logarithmic tree volume ($\text{var}(\log(V)) = 0.00246$) and wood density ($\text{var}(BD) = 688$).

If SW is assumed to be the measured value for stem wood biomass, then it is possible to predict random stand parameters first for the stem wood equation, and then for the equations for other biomass components. The random parameter of the stem wood biomass (u_1) was predicted using Eq. (5). The matrices and vectors needed to predict u_1 are (see Table 4):

$$y_1 = \begin{bmatrix} \ln(SW_{k1}) \\ \dots \\ \ln(SW_{kn}) \end{bmatrix}, \dot{\mathbf{i}}_1 = \begin{bmatrix} -4.879 + 9.651\dfrac{d_{Sk1}}{(d_{Sk1}+12)} + 1.012\ln(h_{k1}) \\ \dots \\ -4.879 + 9.651\dfrac{d_{Skn}}{(d_{Skn}+12)} + 1.012\ln(h_{kn}) \end{bmatrix}, \mathbf{Z} = \begin{bmatrix} 1 \\ \dots \\ 1 \end{bmatrix}$$

$\mathbf{R} = \text{var}(e_{ki})$ $\mathbf{I} = 0.005443 + \text{var}(\log(SW))$, $\mathbf{D} = \text{var}(u_1) = 0.002632$. When the random parameter u_1 is predicted, random stand parameters for stem bark u_2, living branches u_3, dead branches u_4 and total aboveground biomass u_5 can be predicted by applying Eq. (6). The matrices and vectors needed are (see Table 4):

$$\mathbf{T} = \text{cov}\left(\begin{bmatrix} u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix}, (u_1) \right) = \begin{bmatrix} 0.00001 \\ -0.00328 \\ -0.03529 \\ 0.000993 \end{bmatrix}, \dot{\mathbf{i}}_{uj} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{D} = \text{var}(u_1) = 0.002632$$

Table 8. Average fixed (Pred$_{fixed}$) and calibrated predictions (Pred$_{calib}$) for different biomass components obtained by multivariate model (1).

	Plot	Stem wood, kg	Stem bark, kg	Living branches, kg	Dead branches, kg	Total, kg
Measured	4_11	70.8	10.4	9.4	0.1	90.7
Pred$_{fixed}$	4_11	70.9	10.9	11.4	0.7	94.5
Pred$_{calib}$	4_11	70.7	10.9	11.4	0.7	94.5
Measured	357_11	107.6	19.2	29.1	3.5	159.3
Pred$_{fixed}$	357_11	120.3	18.5	25.8	1.4	165.8
Pred$_{calib}$	357_11	115.9	18.5	27.0	2.3	163.5

Eqs. (5) and (6) gave the prediction of the random parameters for stand 4_11; $u_1 = -0.0022$, $u_2 = 0.000$, $u_3 = 0.0027$, $u_4 = 0.0296$ and $u_5 = -0.0008$, and for stand 357_11; $u_1 = -0.0373$, $u_2 = -0.0001$, $u_3 = 0.0465$, $u_4 = 0.4999$ and $u_5 = -0.0141$. The calibrated predictions (Pred$_{calib}$) were obtained by adding the random stand parameter to the fixed prediction (Pred$_{fixed}$) obtained by multi-variate model (1). In stand 4_11, where $v_{d,d6,h}$ equals the volume calculated using d and h ($v_{d,h}$), Pred$_{calib}$ did not differ from Pred$_{fixed}$ (Tables 7 and 8). However, in stand 357_11, where $v_{d,d6,h}$ is lower than $v_{d,h}$, Pred$_{calib}$ gave, on the average, less biased estimates for all tree components, excluding the bark biomass, than Pred$_{fixed}$ (Tables 7 and 8).

5 Discussion

The biomass equations compiled in this study for individual birch trees (*B. pendula* and *B. pubescens*) are applicable over a large part of Finland. However, in the northernmost parts of Finland, in coastal areas and on peatlands, the validity of the functions is uncertain due to the lack of material. The equations are applicable to the whole growing stock, and are valid over a wide diameter range up to 38 cm. The equations were based on variables commonly measured in forest inventories, and were formulated so that the predictions would be logical throughout the range of the material, and even in cases where the functions were extrapolated. The best expression of diameter in the models was d_S/d_S+n

($d_S = 2 + 1.25d$), which tended not to produce an overestimation for large trees and behaved more logically in the extrapolation compared to the generally used transformation for tree diameter, ln(d) (See Marklund 1988).

All the above-ground tree components were relatively well represented in the material, apart from birch foliage. The equation for birch foliage was based on only 21 sample trees, and it is valid over a narrower diameter range from 11 to 26 cm. Similarly, the below-ground biomass equations were based on a relatively deficient material. In addition, the biomass of roots >1 cm was measured on only six sample trees, and for the rest of the root material it was estimated using simple regression. These facts should be kept in mind when applying the model for root biomass, especially for trees with a diameter of under 5 cm or over 25 cm, and for trees growing on peatlands where the root biomass is usually higher than that on mineral soil (Hakkila 1972, Marklund 1988).

The equations were based on subjectively selected experiments and temporary sample plots, concentrated especially in southern Finland. Although the study material was selected from a wide range of stand and site conditions, it was not an objective, representative sample of all the stands in Finland, and this may restrict the generalization and applicability of the equations. The birch material relatively well represented the average tree variables on mineral soil in South Finland, but the growth of the trees in the study material was clearly higher than the average for northern Finland (according to the 9th NFI). Due to the lack of representative material, except for some tree components, the equations based only

on tree diameter are not presented here. Some recommendations for suitable applications of the compiled equations need to be set.

In the model application, there is a risk of systematic prediction error when multivariate model (1) is used. Because multivariate model (1) is based only on tree diameter and height, it can produce biased predictions in populations where the distribution of tree characteristics deviates from that of the study material. This risk can be decreased by applying multivariate models (2) or (3) to stands on mineral soil and peatlands.

For crown components, i.e. living branches and foliage, the most applicable and stable prediction was obtained by using Eqs. (A3) and (A6) based on diameter, height and a crown variable (crown ratio or crown length). The crown variable diminished significantly the between-stand variance, which implies better predictions for the stand level crown biomass. This will also improve the prediction for energy wood stocks, which was the most unreliable when Marklund's (1988) equation based on diameter and height was applied (Kärkkäinen 2005). The crown equations were slightly improved by adding tree age and growth as independent variables, but this also increased the multi-collinearity of the independent variables, which subsequently increased the risk of a biased prediction. Application of multivariate model (1) based only on tree diameter and height can lead to biased predictions for the crown components in northern Finland and on peatlands, especially on undrained mires, where the diameter-height relationship deviates from that of the study material. In these cases a low height to diameter ratio obviously produces overestimates for living branches due to the negative nature of the parameter estimates of tree height in Eq. (9).

The equations for stem wood biomass are valid for trees of all sizes, as well as for trees with a height under 6 m. Eqs. (A1) and (A7) for stem wood biomass included similar independent variables in multivariate models (2) and (3). Because they gave comparable predictions both equations are applicable. The compiled equations for stem wood biomass do not take into account diameter- and height-independent stem form variation, which has a strong influence on tree volume and also on stem biomass (Hakkila 1979). When the

upper diameter, such as the diameter at a height of six meter is measured, the stem biomass can be calculated more reliably by applying an applicable volume function and Eq. (16) for average wood density (Repola et. al 2007). Stem volume can be converted into biomass by multiplying the predicted stem density by the volume.

Equations for the biomass of individual tree components have frequently been estimated separately, i.e. independently, while ignoring the dependence among the biomass components of the same stand or tree. In this study the across-equation correlation was studied at both the stand and tree level because the measurements of the biomass components were based on the same sample trees. In the analysis the assumed statistical dependence between biomass components was verified especially at the stand level. Based on this finding, there were therefore justifiable reasons to use a multivariate modelling approach in the analysis. First, the multivariate procedure produces more reliable parameter estimates than when the equations are estimated independently (Parresol 1999). In our study, however, this advantage was only minor because the multivariate procedure only slightly changed the parameter estimates (See Repola et al. 2007), and it gave only slightly more reliable predictions than those obtained by independently estimated equations. Second, by utilizing the across-equation covariance of the random parameters at the stand level especially, it is possible to carry information from one equation to other equation (Lappi 1991). In the model application, the fixed prediction of a biomass component can be calibrated to a given stand by utilizing the across-equation covariance of random stand parameters and measurements of other biomass component (See Lappi 1991). The measurement of one biomass component results in more reliable predictions for the other biomass components. This model calibration is especially applicable for research purposes when, with the help of few biomass measurements, it is possible to obtain more reliable biomass estimates for all the trees in a stand. This is a good alternative compared to collecting new biomass data for model construction.

In multivariate modelling, biomass additivity can be ensured by setting across-equation constraints and by constructing the total tree equation

such that it is a function of all the independent variables used in the tree component equations (Parresol 1999). In order to avoid a complex total tree equation with several transformations of the independent variables (height and diameter), and to simplify the model structure, the additivity procedure was not applied. Despite this, the summed predictions of the tree components corresponded relatively well, on the average, to the prediction for the whole tree obtained by applying the constructed multivariate models. To ensure the biomass additivity as presented in the results, only the equations in the same multivariate model should be applied.

Determination of the dependent variables (biomass of tree components or total tree biomass) was not based on direct measurements, but on sub-samples. This process of determining the biomass introduces an error in the biomass estimates (Paressol 1999, 2001). When assessing the reliability of the predicted biomass value, statistical errors in the dependent variable caused by the sub-sampling should also not be ignored (Paressol 1999). Because this source of statistical error could not be estimated reliably, it was not taken into account in the model estimation and no estimate for the magnitude of this error was presented.

The constructed biomass equations are applicable for a wide range of stand and site conditions in Finland. Due to the applied statistical method the equations produce more reliable biomass estimates compared to the equations presented earlier (Repola et al. 2007). Furthermore, the structure of the multivariate models enables more flexible application of the equations, especially for research purposes.

Acknowledgements

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Appendix

Multivariate Model 2

Above-ground biomass equations (Table A1):

Stem wood:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 12)} + b_2 \ln(h_{ki}) + b_3 \frac{d_{ki}}{t_{13ki}} + u_k + e_{ki} \tag{A1}$$

Stem bark:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 12)} + b_2 \frac{h_{ki}}{(h_{ki} + 20)} + u_k + e_{ki} \tag{A2}$$

Living branches:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 12)} + b_2 \frac{h_{ki}}{(h_{ki} + 12)} + b_3 cl_{ki} + u_k + e_{ki} \tag{A3}$$

Dead branches:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 16)} + u_k + e_{ki} \tag{A4}$$

Total (above-ground):

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 12)} + b_2 \frac{h_{ki}}{(h_{ki} + 22)} + b_3 t_{13ki} + u_k + e_{ki} \tag{A5}$$

Separate Model 2

Foliage:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 2)} + b_2 cr_{ki} + u_k + e_{ki} \tag{A6}$$

Multivariate Model 3

Above-ground biomass equations (Table A2):

Stem wood:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 12)} + b_2 \ln(h_{ki}) + b_3 \frac{d_{ki}}{t_{13ki}} + u_k + e_{ki} \tag{A7}$$

Stem bark:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 8)} + b_2 \frac{h_{ki}}{(h_{ki} + 22)} + b_3 \ln(bt_{ki}) + u_k + e_{ki} \tag{A8}$$

Living branches:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 10)} + b_2 \frac{h_{ki}}{(h_{ki} + 10)} + b_3 \ln(i_{5ki}) + b_4 cl_{ki}$$
$$+ b_5 t_{13ki} + u_k + e_{ki} \tag{A9}$$

Dead branches:

$$\ln(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 6)} + b_2 \frac{h_{ki}}{(h_{ki} + 10)} + b_3 t_{13ki} + b_4 i_{5ki} + u_k + e_{ki} \qquad (A10)$$

Total (above-ground):

$$(y_{ki}) = b_0 + b_1 \frac{d_{Ski}}{(d_{Ski} + 12)} + b_2 \frac{h_{ki}}{(h_{ki} + 22)} + b_3 i_{5ki} + b_3 \frac{d_{ki}}{t_{13ki}} + u_k + e_{ki} \qquad (A11)$$

where
y_{ki} = biomass component or total biomass of tree i in stand k, kg
d_{ki} = tree diameter at breast height of tree i in stand k, cm
d_{Ski} = $2 + 1.25\,d$, cm
h_{ki} = tree height of tree i in stand k, m
cl_{ki} = length of living crown of tree i in stand k, m
cr_{ki} = crown ratio of tree i in stand k, ($cr_{ki} = cl_{ki}/h_{ki}$)
t_{13ki} = tree age at breast height of tree i in stand k
bt_{ki} = double bark thickness at breast height of tree i in stand k, cm
i_{5ki} = breast height radial increment during the last five years of tree i in stand
$\quad$ k, mm

Table A1. The parameter estimates of multivariate model 2. For the fixed parameters the standard error is given in parentheses. Variances and covariances of random stand parameters (u_{nk}) and residual errors (e_{nki}), and the empirical correction factor (c) for dead branches models, are given.

	Stem wood Eq. (A1)	Stem bark Eq.(A2)	Living branches Eq. (A3)	Dead branches Eq. (A4)	Total above-ground Eq. (A5)	Foliage Eq. (A6)
Fixed	N=127	N=127	N=127	N=127	N=127	N=21
b_0	−4.886	−5.433	−5.067	−7.996	−3.659	−20.856
	(0.058)	(0.152)	(0.183)	(1.141)	(0.053)	(4.015)
b_1	9.965	10.121	14.614	11.824	10.588	22.320
	(0.164)	(0.467)	(0.580)	(1.966)	(0.157)	(4.628)
b_2	0.966	2.647	−5.074		2.966	2.819
	(0.040)	(0.509)	(0.563)		(0.159)	(0.795)
b_3	−0.135		0.092		0.0006	
	(0.025)		(0.009)		(0.0002)	
Random	u_{1k}	u_{2k}	u_{3k}	u_{4k}	u_{5k}	u_{6k}
u_{1k}	0.00160					
u_{2k}	−0.00093	0.01059				
u_{3k}	−0.00167	0.00593	0.01508			
u_{4k}	−0.02431	−0.04852	−0.02610	1.0650		
u_{5k}	0.00065	0.00063	0.00128	−0.02471	0.00049	
u_{6k}						0.0108
	e_{1ki}	e_{2ki}	e_{3ki}	e_{4ki}	e_{5ki}	e_{6ki}
e_{1ki}	0.00537					
e_{2ki}	0.00436	0.04419				
e_{3ki}	0.00723	0.00081	0.05663			
e_{4ki}	0.00456	−0.01444	−0.03226	2.6908		
e_{5ki}	0.00528	0.00876	0.01256	0.00504	0.00711	
e_{6ki}						0.044
c				2.1491		

Table A2. The parameter estimates of multivariate model 3. For the fixed parameters the standard error is given in parentheses. Variances and covariances of random stand parameters (u_{nk}) and residual errors (e_{nki}), and the empirical correction factor (c) for dead branches models, are given.

	Stem wood	Stem bark	Living branches	Dead branches	Total above-ground
	Eq. (A7)	Eq. (A8)	Eq. (A9)	Eq. (A10)	Eq. (A11)
Fixed	$N=124$	$N=124$	$N=124$	$N=124$	$N=124$
b_0	−4.915	−5.304	−5.918	−16.113	−3.713
	(0.058)	(0.303)	(0.193)	(1.983)	(0.050)
b_1	9.984	8.498	12.867	37.902	10.616
	(0.174)	(0.591)	(0.612)	(5.801)	(0.155)
b_2	0.981	3.380	−3.573	−17.342	3.235
	(0.042)	(0.511)	(0.571)	(4.654)	(0.164)
b_3	−0.180	0.382	0.238	−0.063	0.007
	(0.043)	(0.057)	(0.035)	(0.013)	(0.001)
b_4			0.095	−0.166	−0.214
			(0.010)	(0.041)	(0.039)
b_5			0.007		
			(0.001)		
Random	u_{1k}	u_{2k}	u_{3k}	u_{4k}	u_{5k}
u_{1k}	0.0014				
u_{2k}	−0.00076	0.01135			
u_{3k}	−0.00304	0.00371	0.01171		
u_{4k}	−0.01823	0.01111	0.04410	0.5783	
u_{5k}	0.00016	−0.00021	0.00017	−0.00569	$4.92 \cdot 10^{-38}$
	e_{1ki}	e_{2ki}	e_{3ki}	e_{4ki}	e_{5ki}
e_{1ki}	0.00534				
e_{2ki}	0.00543	0.03508			
e_{3ki}	0.00576	0.00386	0.04300		
e_{4ki}	0.01639	0.00368	0.02686	2.5697	
e_{5ki}	0.00518	0.00891	0.00984	0.02571	0.00673
c				1.788	