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Models for Vertical Wood Density of Scots Pine, Norway Spruce and Birch Stems, and Their Application to Determine Average Wood Density

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The purpose of this study was to investigate the vertical dependence of the basic density of Scots pine, Norway spruce, and birch stems, and how such dependence could be applied for determining the average stem wood density. The study material consisted of 38 Scots pine (*Pinus sylvestris*), 39 Norway spruce (*Picea abies* [L.] Karst.) and 15 birch (*Betula pendula* and *Betula pubescens*) stands located on mineral soil sites in southern Finland. The stem material mainly represented thinning removal from stands at different stages of development. The linear mixed model technique, with both fixed and random effects, was used to estimate the model.

According to the fixed part of the model, wood density was dependent on the vertical location along the stem in all three tree species. Wood density in pine decreased from the butt to the top, and the gradient in wood density was steep at the butt but decreased in the upper part of the stem. The vertical dependence was similar in birch, but the density gradient was much smaller. For spruce the vertical dependence of the basic density was moderate.

The model can be calibrated for a tree stem when one or more sample disks are measured at freely selected heights. Using treewise calibrated predictions of the vertical density dependence and measured stem diameters, almost unbiased estimates, and lower prediction errors than with traditional methods, were obtained for the average stem wood density. The advantages of the method were greater for pine with a strong vertical dependence in basic density, than for spruce and birch.

Keywords Norway spruce, Scots pine, downy birch, silver birch, wood density, mixed models
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1 Introduction

Since the volume of the growing stock is the main interest in growing timber, stem volume studies have a long tradition of publishing volume tables and volume functions for both practical and scientific purposes. Interest in stem quality and the variation in wood properties have increased during the last couple of decades. However, stem volume is not the most appropriate indicator of quality. Wood density is commonly used to describe the wood quality in the mechanical and chemical wood industry. There are many reasons for this interest in wood density as it is correlated with a number of wood quality properties.

Information about the wood density is also needed in stem biomass estimation. In many studies, stem biomass (dry weight) has been determined by multiplying the stem volume by the average stem wood density, with several modifications (Hakkila 1979, Pulkkinen and Pöykkö 1990, Mäkinen 1996, Naidu et al., 1998, Mäkelä and Vanninen 1998, Vanninen and Mäkelä 2000). The reliability of estimates for stem biomass depends on how accurately the stem volume and average wood density have been estimated.

Many methods have been used to estimate the average wood density of the stem. In Finland, Hakkila (1979) constructed a regression equation to predict the average stem density of pine, spruce and birch on the basis of the measured density (increment core) at breast height and measured external tree characteristics, tree age, and the growth rate. The density of pine, spruce and birch stem wood at 25% height is approximately the same as the average stem density (Nylinder 1961, Hakkila 1979). In biomass studies, the average stem or log density has been most commonly derived from sample disks taken at different heights along the stem, and by calculating the mean basic density of the wood in the disks, weighted by the disk area. A constant wood density for all stems has also been used to convert stem volume into stem biomass (Bartelink 1996).

The average wood density of the stem is difficult to measure or estimate for many reasons. The average wood density of a stem is affected by a large number of factors such as tree species, geographical location and other environmental factors, site quality, position of the tree in a stand, tree age and size, growth rate and genetic factors (Hakkila 1966, 1979, Uusvaara 1974). Wood density also varies in the radial and vertical directions of the stem according to a species-specific pattern. According to Knigge and Shultz (1966), wood density of most conifers decreases from the butt to the top. The vertical dependence in pine is clear, with a rapid decrease at the stem base but less conspicuous one in the upper portion of the stem (Tamminen 1962, Hakkila 1966, Uusvaara 1974, Hakkila 1979, Björklund 1984). Spruce shows only a slight vertical dependence in wood density, and relatively conflicting results have been reported (Kärkkäinen 1976, Hakkila 1966, 1979, Johansson 1993, Berg et al., 1995). The vertical dependence in birch is the same as that of pine, but the density gradient is much smaller (Hakkila 1966, 1979).

In many studies the average stem density has been derived from measurements made on sample disks. The accuracy of the average stem density estimates is influenced by the number of disks used, and the height along the stem from where the disks are taken. To avoid systematic errors, the vertical dependence of wood density should be known and must be taken into account when cutting sample disks. This is especially important for tree species with a clear vertical dependence of the wood density. For example, the density at breast height is systematically higher than the average stem density of pine, spruce and birch (Hakkila 1979).

When only a few sample disks have been taken from the stem, the accuracy of the wood density estimate can be improved if tree-specific information about the vertical dependence of stem wood density is available. This requires a model describing the vertical dependence of density, and it can be calibrated for a new tree by using measurements of wood density made on sample disks. The risk of systematic errors in the average wood density estimates can be decreased by using calibrated density predictions. Unfortunately, despite the long tradition in wood density research in Finland, only a few models have been published for the vertical variation in density (Björklund and Ferm 1984, Varjo 1991).

Within- and between-tree variation in vertical wood density can be analyzed by linear mixed model techniques. The fixed effect describes the population mean, in our case the average, tree-specific vertical dependence of wood density. In applications, the fixed prediction can be calibrated to new trees with the help of sample disk measurements by predicting the random parameter using linear prediction theory.

The purpose of this study is to construct a model for the vertical dependence of the basic density of Scots pine, Norway spruce, and birch stems. It should be possible to calibrate the model to new trees by utilizing sample disk measurements of wood density, and the model should also be applicable for stem biomass estimation. The linear mixed model technique with both fixed and random parameters was used in the model estimation.

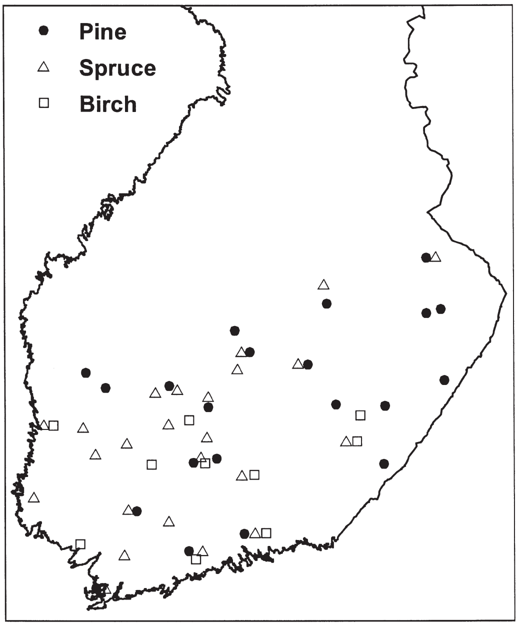

Fig. 1. The location of the study stands. One symbol can represent more than one stand.

2 Material and Methods

2.1 Stands

The basic density material was obtained from sample trees collected during 1993–2000 in connection with the National Bioenergy Research Program, carried out by the Finnish Forest Research Institute. The material consisted of a total of 92 stands, comprising 38 Scots pine (*Pinus sylvestris*) stands, 39 Norway spruce (*Picea abies* [L.] Karst.) stands and 13 birch stands (*Betula pendula* and *Betula pubescens*). All the stands were located on mineral soil sites in southern Finland (Fig. 1). The stands had previously been managed according to normal forestry practices. The distribution of forest site types was typical for Scots pine, Norway spruce and birch on mineral soil (Table 2). The spruce and birch stands represented the mesic *Myrtillus* type (MT) and the fertile *Oxalis-Myrtillus* type (OMT) (Cajander 1949). The Scots pine stands represented the dryish *Vaccinium* type (VT), the mesic *Myrtillus* type (MT), and dry *Calluna* type (CT).

2.2 Sample Trees

The study material consisted of the thinning removal from stands at different stages of development; first and second commercial thinning, and final cutting (Table 1). The birch material was collected only in stands at the first commercial thinning. The sample trees from thinning stands mainly represented trees removed in thinning from below. The trees from final cutting represent the whole growing stock.

From each stand 15 sample trees, nearest to the centre point of the plot, were felled. In thinning stands, ten trees were selected from the thinning removal and five trees mainly from along the haulage trail. Damaged trees were not accepted as sample trees. The total number of sample trees was 1365: 585 pine, 585 spruce and 195 birch (Table 1). The birch material consisted of both downy birch (*Betula pubencens*) and silver birch (*Betula pendula*). Most of the sample trees were small diameter stems (Figs. 2 and 3).

2.3 Field Work and Sample Disks

The sample trees were felled and the stem dimensions, i.e. height, diameter at breast height, and

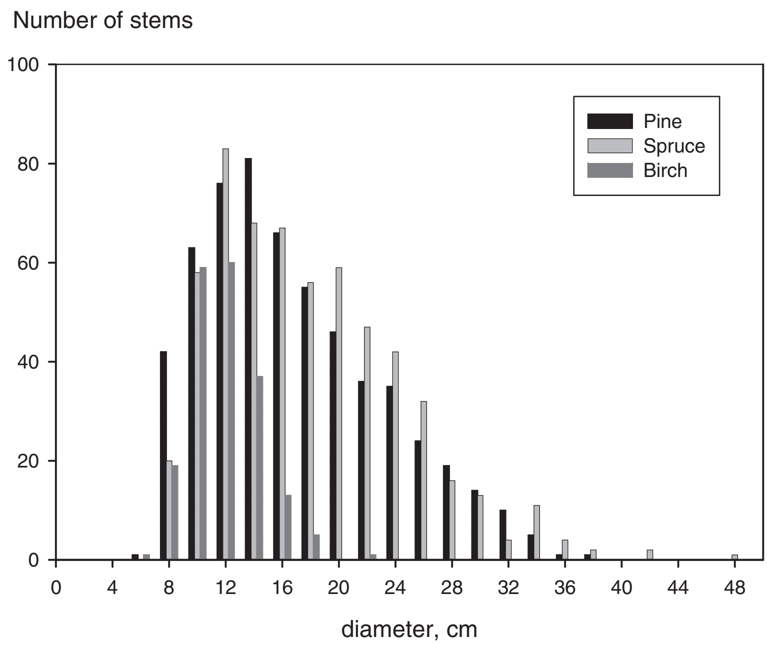

Fig. 2. The diameter distribution of the sample trees by tree species.

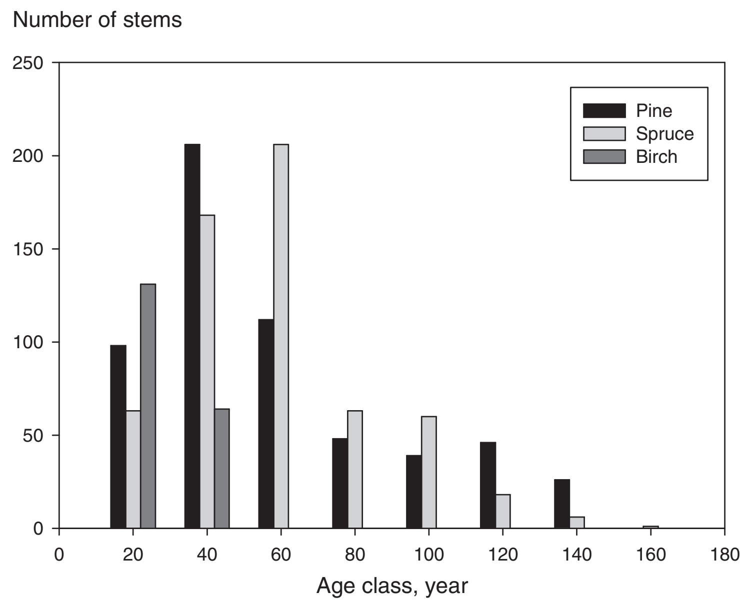

Fig. 3. The age distribution of the sample trees by tree species.

Table 1. Characteristics of the study material.

Stage of development	Number of stands	Number of sample trees	Number of disks	Age, years	D, cm	H, m	G, $m^2 ha^{-1}$
First thinning							
Pine	15	225	2545	34	12.7	11.8	26.5
Spruce	15	240	3008	40	13.7	13.3	29.4
Birch	13	195	2847	30	12.7	16	24.8
Second thinning							
Pine	16	240	3456	61	18.6	18.1	29
Spruce	15	225	3456	61	20.6	21.2	32.9
Final cutting							
Pine	7	120	1256	115	27.8	23.1	25.6
Spruce	8	120	1566	101	28.2	24.2	30.9

height at the crown base, were measured. 2- to 3-cm thick, knot-free sample disks were taken mainly at one-meter intervals from the butt to the top of the stem. The sample disks were cut at one-meter intervals from the pulpwood and unmerchantable top sections, and from the saw logs at the base and top of the logs (4- to 5-meter intervals). The number of disks per tree varied from 7 to 20 (Table 1). The disks were stored in plastic bags and taken to the laboratory for measurement of disk fresh weight and dimensions. The disks were separated into wood and bark sections and, after 2–3 days drying at a temperature of 106 °C, the basic density of the wood was determined as the dry matter weight per unit volume of green wood.

2.4 Statistical Analysis

The mixed model technique was used to fit the models for the vertical dependence of the basic density of the stem. The material had a hierarchical structure, with stand, tree and within-stem levels. In the model, the stand and tree level effects were combined at the tree level because the stand level was not significant when tree growth rate, which correlated with site quality, was used as independent variable. The final model structure was:

$$\hat{\mathbf{y}}_{ik} = \mathbf{x}_{ik}^T \mathbf{b} + \mathbf{z}_{ik}^T \mathbf{u}_\mathrm{k} + \mathbf{e}_{ik} \tag{1}$$

where
$\mathbf{y}_{ik}$ = basic density at stem position i in tree k
$\mathbf{x}_{ik}$ = vector of the fixed regressors for position i in tree k
$\mathbf{b}$ = vector of the fixed effects
$\mathbf{z}_{ik}$ = vector of the independent random regressors for tree k
$\mathbf{u}_k$ = vector of the random effects for tree k
$\mathbf{e}_{ik}$ = random error term for position i in tree k

The best linear estimates for both **b** and **u** can be found simultaneously from the equation (Searle 1987):

$$\begin{bmatrix} \mathbf{X}^T\mathbf{R}^{-1}\mathbf{X} & \mathbf{X}^T\mathbf{R}^{-1}\mathbf{Z} \\ \mathbf{Z}^T\mathbf{R}^{-1}\mathbf{X} & \mathbf{Z}^T\mathbf{R}^{-1}\mathbf{Z}+\mathbf{D}^{-1} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{b}} \\ \hat{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}^T\mathbf{R}^{-1}\mathbf{y} \\ \mathbf{Z}^T\mathbf{R}^{-1}\mathbf{y} \end{bmatrix} \tag{2}$$

where
$\mathbf{X}$ = $N \times q$ matrix of the fixed regressors
N = number of sample disks in the whole material
$\mathbf{Z}$ = $N \times p$ matrix of the random regressors, where N is the number of sample trees and p is the number of random variables
$\hat{\mathbf{u}}$ = vector of the random effects
$\hat{\mathbf{b}}$ = vector of the fixed effects
$\mathbf{y}$ = vector of the independent variable, length N
$\mathbf{R}$ = var(e), $N \times N$ diagonal matrix
$\mathbf{D}$ = var(u), covariance matrix of the tree level random effects

The mixed linear model technique was used to construct the models by species from the whole material. The birch model was based on both birch

Table 2. The number of study stands by forest site type.

	Forest site type			
	Dry (Cl)[a)]	Dryish (VT)[a)]	Mesic (MT)[a)]	Fertile (OMT)[a)]
Pine stands	4	23	11	-
Spruce stands	-	-	25	13
Birch stands	-	-	9	4

[a)] CT = dry Calluna type, VT = dryish Vaccinium type, MT = mesic Myrtillus type, OMT = fertile Oxalis-Myrtillus type.

species (*Betula pendula* and *Betula pubescens*). The unstructured covariance structure was used as an assumption for random effects. The models were fitted with MLwiN software version 2.1a.

In the application phase, the compiled equations were used to determined average stem wood density. First the mixed model was calibrated for each tree by predicting the random parameters using the sample disk measurements. It is assumed that the random parameters of the same tree are correlated, and therefore this information can be used in the calibration. The random vector ($\hat{\mathbf{u}}_k$) of the tree can be predicted as follows (Searle 1987, Lappi 1991, McCulloch and Searle 2001):

$$\hat{\mathbf{u}}_k = (\mathbf{Z}^T\mathbf{R}^{-1}\mathbf{Z} + \mathbf{D}^{-1})^{-1}\mathbf{Z}^T\mathbf{R}^{-1}(\mathbf{y}_k - \grave{\mathbf{i}}_k) \tag{3}$$

where
$\hat{\mathbf{u}}_k$ = vector of the random effects for tree k
$\mathbf{Z}$ = model matrix of the random effects in tree k
$\mathbf{R}$ = var(e) = matrix of the random term for stem position i in tree k
$\mathbf{D}$ = covariance matrix of the random parameters
$\grave{\mathbf{i}}_k$ = vector of the fixed effects (prediction of the fixed part of the model)
$\mathbf{y}_k$ = observed random vector in tree k

3 Results

3.1 Models for the Vertical Dependence of the Basic Density of the Stem

In the analyses, the stand level variance was not significant and only tree and within-tree levels were addressed in the final models. In the fixed part of the models, the growth rate, location and stem dimension were used as independent variables (Table 3). Location along the stem was expressed as the relative height, and ranged from 0 (butt) to 1 (top). The interaction between the tree diameter at breast height and tree age (diameter divided by age), which indicates the growth rate, improved the equation considerably for all tree species. The random part of the model, indicating the tree level variation in the density gradient in the vertical direction, consisted of constant (u_{0k}) and random coefficients of relative height (u_{1k} and u_{2k}).

Pine:

$$\begin{aligned}\mathrm{BD}_{ik} &= b_0 + b_1 h_k + b_2 \frac{d_k}{t_k} + b_3 hr_{ik} + b_4 hr_{ik}^3 \\ &\quad + b_5 hr_{ik}^5 + b_6 h_k hr_{ik} + b_7 h_k^2 hr_{ik} + u_{0k} \\ &\quad + u_{1k} hr_{ik} + u_{2k} hr_{ik}^2 + e_{ik}\end{aligned} \tag{4}$$

Spruce:

$$\begin{aligned}\mathrm{BD}_{ik} &= b_0 + b_1 h_k + b_2 \frac{d_k}{t_k} + b_3 hr_{ik} + b_4 hr_{ik}^2 \\ &\quad + u_{0k} + u_{1k} hr_{ik} + u_{2k} hr_{ik}^2 + e_{ik}\end{aligned} \tag{5}$$

Birch:

$$\begin{aligned}\mathrm{BD}_{ik} &= b_0 + b_1 h_k + b_2 \frac{d_k}{t_k} + b_3 hr_{ik} + b_4 hr_{ik}^2 \\ &\quad + b_5 hr_{ik}^3 + u_{0k} + u_{1k} hr_{ik} + u_{2k} hr_{ik}^2 + e_{ik}\end{aligned} \tag{6}$$

where
BD_{ik} = basic density of stem position i in tree k, kg/m^3
d_k = tree diameter at breast height, mm
t_k = tree age, years
h_k = tree height, dm
hr_{ik} = relative height of position i in tree k
$b_0 \ldots b_8$ = parameters of the fixed effects
u_{0k}, u_{1k} and u_{2k}
= parameters of the random effects in tree k
e_{ik} = random parameter of stem position i in tree k

The expected wood density of the three species depended on the vertical location (Fig. 4). Pine and birch density decreased from the butt to the top. For pine, the density gradient was steep at

Table 3. Parameter values of models (4–6) for the vertical dependence of the basic density of the stem. For the fixed parameters, the standard error of estimates is given in parentheses.

Parameter	Scots pine (3) Estimate	Norway spruce (4) Estimate	Birch (5) Estimate
b_0	467.073 (5.490)	431.339 (4.639)	470.098 (13.980)
b_1	0.228 (0.026)	0.075 (0.018)	0.359 (0.078)
b_2	–11.777 (0.829)	–16.973 (1.041)	–9.612 (1.750)
b_3	–239.074 (11.999)	–41.801 (3.782)	–82.642 (8.915)
b_4	332.810 (6.502)	63.583 (4.150)	180.597 (18.632)
b_5	–231.238 (4.352)		–136.931 (12.335)
b_6	0.346 (0.135)		
b_7	–0.001 (0.000)		
var(u_{0k})	1068.687	637.196	799.304
var(u_{1k})	9496.149	6664.309	4760.085
var(u_{2k})	8177.287	8398.160	5036.492
cov(u_{0k},u_{1k})	–2136.378	–324.087	–662.301
cov(u_{0k},u_{2k})	1430.737	–43.835	317.603
cov(u_{1k},u_{2k})	–8275.861	–6931.747	–4591.453
e_{ik}	92.740	114.407	115.461

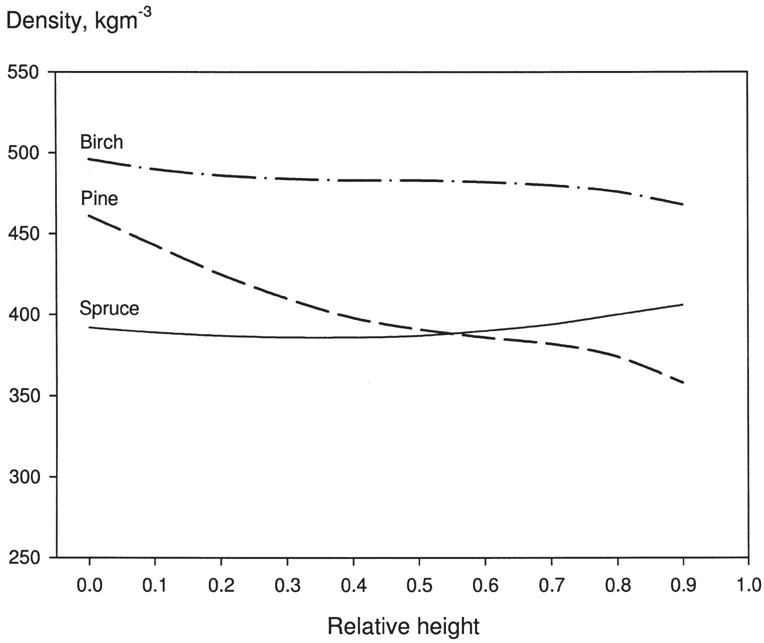

Fig. 4. Example of fixed predictions of wood density in the vertical direction along birch, pine and spruce stems according to models 4–6 (h = 18 m and d_k/t_k = 4).

the butt and decreased in the upper part of the stem. The difference in basic density between the but and the top was considerable, about 100 kg/m^3. The vertical dependence trend for birch was similar, but the density gradient was much flatter. For spruce, the vertical dependence of the basic density was slight; the density first decreased slowly and then started to increase when approaching the top.

3.2 Application of the Models for Determining Average Wood Density

Prediction of the average wood density of birch, pine and spruce stems was investigated on the trees from which sample disks had been cut at one-meter intervals. This sub-data included pine, spruce and birch stems with diameters varying from 7 to 20 cm (on average 12–13 cm). First, the average wood density was calculated using all the available density information ($BD_{measured}$). $BD_{measured}$, called measured density here, is the average wood density of the stem calculated as the basal area-weighted mean of all the sample disks. Second, one or two sample disks (at breast height and at a relative height of 70%) per tree were used to predict the average wood density in two ways:

1) The average wood density was determined using only the measured density of the sample disks ($BD_{1.3m}$ and $BD_{1.3m\_0.7h}$). $BD_{1.3m}$ is the wood density of the stem at breast height, and $BD_{1.3m\_0.7h}$ is the average wood density of the stem calculated as the basal area-weighted mean of two sample disks (at breast height and at a relative height of 70%).
2) The average wood density was determined by applying Eqs. 4–6, and the sample disks measurements ($BD_{model(1.3m)}$, $BD_{model(1.3m\_0.7h)}$) and measured diameters at one-meter intervals. $BD_{model(1.3m)}$ is the prediction for the average wood density, which was calibrated using density information obtained at breast height. $BD_{model(1.3m\_0.7h)}$ is the prediction calibrated with the density information at breast height and at a relative height of 70%. Eqs. 4–6 were first calibrated for any tree by predicting the random vector ($\hat{\mathbf{u}}_k$) using the density measurement made on the sample disks. The random vector ($\hat{\mathbf{u}}_k$) of the tree can be predicted using Eq. 3. The matrices and vectors needed for predicting the random parameters (u_{0k} , u_{1k} and u_{2k}) of birch Eq. 6 using two sample disks, are:

Vector of the random parameter

$$\hat{\mathbf{u}}_k = \begin{bmatrix} u_{0k} \\ u_{1k} \\ u_{2k} \end{bmatrix}$$

Measured basic density (BD) of the sample disks of tree k

$$\mathbf{y}_k = \begin{bmatrix} BD_{k1} \\ BD_{k2} \end{bmatrix}$$

Fixed prediction of the model

$$\grave{\mathbf{\imath}}_k = \begin{bmatrix} b_0 + b_1 h_{k1} + ... + b_5 hr_{k1}^3 \\ b_0 + b_1 h_{k2} + ... + b_5 hr_{k2}^3 \end{bmatrix}$$

Model matrix of the random parameter of tree k

$$\mathbf{Z} = \begin{bmatrix} 1 & hr_{k1} & hr_{k1}^2 \\ 1 & hr_{k2} & hr_{k2}^2 \end{bmatrix}$$

Random error

$$\mathbf{R} = \mathrm{var}(e_{ik})\,\mathbf{I} = 115.561\,\mathbf{I}$$

Covariance matrix of the random parameters

$$\mathbf{D} = \mathrm{var}\begin{bmatrix} u_{0k} \\ u_{1k} \\ u_{2k} \end{bmatrix} = \begin{bmatrix} 799.304 & -622.301 & 317.603 \\ -662.301 & 4760.085 & -4591.453 \\ 317.603 & -4591.453 & 5036.492 \end{bmatrix}$$

The prediction of the fixed part of the model deviated from the measured density value of the tree (Fig. 5). By calibrating the fixed prediction using random parameters (intercept and slope), the vertical dependence was predicted more accurately especially in the lower part of the stem (Fig. 5). Using density information at a relative height of 70%, in addition to breast height, the prediction of the upper part of the stem became more accurate.

The average wood density based only on the measured density of the sample disks ($BD_{1.3m}$

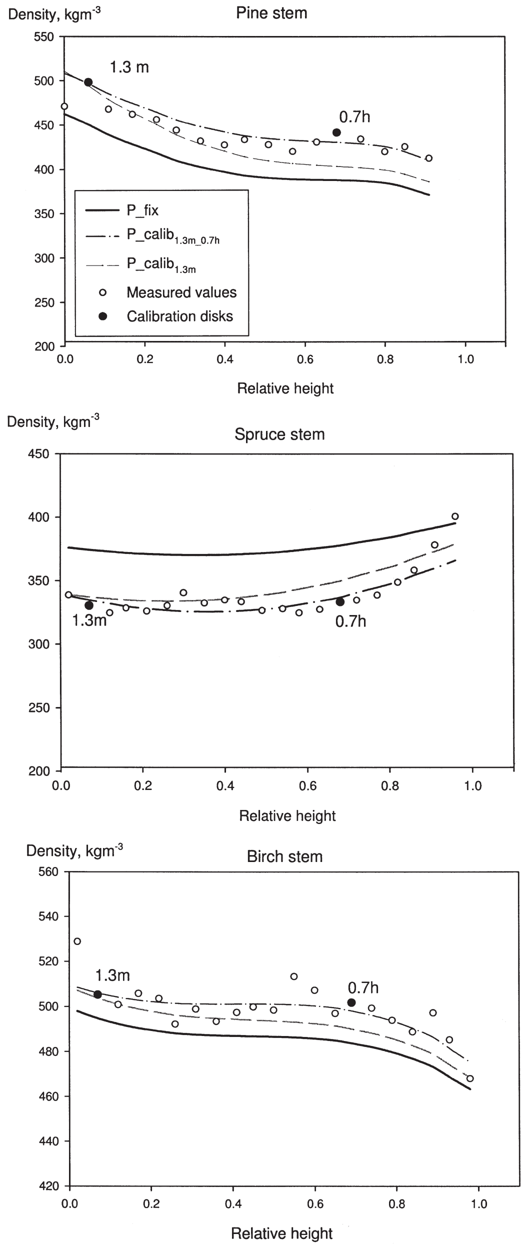

Fig. 5. Example of the predicted wood density in the vertical direction along birch, pine and spruce stems. P_fix are the predictions of the fixed part of Eqs. 4, 5 or 6. $P\_calib_{1.3m\_0.7h}$ and $P\_calib_{1.3m}$ are the predictions (Eqs. 4, 5 or 6), which have been calibrated with random parameters using one or two sample disk measurements.

and $BD_{1.3m\_0.7h}$) resulted in overestimates of the stem wood density for all tree species (Table 4). The overestimate was low, under 1.0%, for the spruce and birch stems, but the average prediction error was clearly larger, –2.3% and –5.3%, for the pine stems (Table 4). Using two sample disks ($BD_{1.3m\_0.7h}$) instead of one sample disk ($BD_{1.3m}$) resulted in more accurate results, and a decrease in the average prediction error and standard deviation of the prediction errors.

The average wood density determined on the basis of the measured density of sample disks, the equations and measured diameters ($BD_{model(1.3m)}$, $BD_{model(1.3m\_0.7h)}$), gave predictions with a prediction error of under 1.0% for the three tree species. Using two sample disks ($BD_{model(1.3m\_0.7h)}$) in the model calibration instead of one ($BD_{model(1.3m)}$) did not significantly affect the average prediction error, but it did reduce the standard deviation of the prediction error (Table 4). The use of $BD_{model(1.3m)}$ and $BD_{model(1.3m\_0.7h)}$ resulted in more reliable estimates for the average wood density compared to $BD_{1.3m}$ and $BD_{1.3m\_0.7h}$. The advantage in using the models, in addition to sample disk measurements, was minor for spruce stems, but considerable for pine stems.

4 Discussion

In this study the vertical dependence of the wood density of pine, spruce and birch stems was analyzed using a linear mixed model. The aim was to construct a model that can be calibrated for any stem using one or more wood density measurements at a freely chosen height, and which can be used to determine the average wood density of a stem.

The fixed predictions of the models delineated the average vertical dependence of the wood density. In this study, wood density in Scots pine dereased sharply from the butt to the top and the difference in wood density between the butt and top in pine stems was over 100 kg/m^3. The result is in agreement with the earlier studies on Scots pine and also other pine species (Tamminen 1962, Hakkila 1966, Knigge and Shultz 1966, Uusvaara 1974, Mette and Stephan 1976, Hakkila 1979, Barse and Laidly 1980, Björklund

Table 4. The average wood density and prediction errors of the stem (kgm^{-3}) when applying different methods. Standard deviation in parentheses.

Method	Density, kgm^{-3}	Prediction error, kgm^{-3}	Prediction error, %
Pine (N = 327)			
$BD_{measured}$	412.6 (32.7)	-	
$BD_{model(1.3.m)}$	412.2 (31.7)	0.4 (9.1)	0.06 (2.2)
$BD_{model(1.3m\_0.7h)}$	410.1 (31.9)	2.4 (6.3)	0.6 (1.9)
$BD_{1.3m}$	435.0 (42.2)	–22.3 (14.5)	–5.3 (3.3)
$BD_{1.3m\_0.7h}$	422.7 (37.9)	–9.9 (9.8)	–2.3 (2.3)
Spruce (N = 317)			
$BD_{measured}$	385.3 (34.1)	-	
$BD_{model(1.3.m)}$	385.8 (31.8)	–0.5 (8.4)	–0.2 (2.2)
$BD_{model(1.3m\_0.7h)}$	384.6 (32.2)	0.8 (5.6)	0.1 (1.5)
$BD_{1.3m}$	386.2 (36.8)	–1.0 (8.7)	–0.2 (2.2)
$BD_{1.3m\_0.7h}$	386.2 (35.0)	–1.0 (5.7)	–0.3 (1.5)
Birch (N = 192)			
$BD_{measured}$	475.0 (29.5)	-	
$BD_{model(1.3.m)}$	474.7 (26.5)	0.3 (8.3)	0.01 (1.7)
$BD_{model(1.3m\_0.7h)}$	475.0 (38.0)	0.1 (6.9)	0.01 (1.5)
$BD_{1.3m}$	478.3 (32.0)	–3.2 (9.0)	–0.7 (1.9)
$BD_{1.3m\_0.7h}$	476.7 (30.7)	–1.6 (6.9)	–0.3 (1.5)

$BD_{measured}$ is the measured stem density using all the sample disks along the stem, $BD_{1.3m}$ is the density of the stem at breast height, $BD_{1.3m\_0.7h}$ is the average wood density of the stem calculated as the weighted mean of two sample disks (at breast height and at a relative height of 70%), $BD_{model(1.3m)}$ is the average wood density of the stem obtained with Eqs. 3–5 using density information at breast height and measured diameters, $BD_{model(1.3m\_0.7h)}$ is the average wood density of the stem obtained with Eqs. 3–5 using density information at breast height and at a relative height of 70% and measured diameters.

1984). The vertical dependence in birch stems was similar to that in pine, but the density gradient was much flatter, as has also been reported by Jalava (1946), Kujala (1946), Hakkila (1966, 1979) and Björklund (1984). The wood density of silver birch (Betula pendula) and downy birch (Betula pubescens) did not differ when the growth rate was added as an independent variable in the model. Hakkila (1966) obtained a higher wood density for silver birch than for downy birch. The vertical dependence of wood density in spruce was low and more irregular, first decreasing and then increasing again towards the top, which was agreement with the findings for Norway spruce (Hakkila 1979, Frimpong-Mensah 1987) and other spruce species (Spurr et al. 1954, Wahlgren et al. 1966, Heger 1974). A different vertical dependence has also been found (Kärkkäinen 1976, Johansson 1993).

The different results obtained for the vertical dependence, especially in the upper part of the stem, may be due to how far up the stem the last sample disks were taken. For example, Hakkila (1966) and Uusvaara (1974) took the last sample at the height of 80%. In our study the last sample disk was taken at the height of 90–99%. Furthermore, in our material the sample disks of larger-sized stems were taken mainly from the butt and top of the logs, which could lead to unreliability in the resulting wood density, especially in the case of pine stems where the wood density decreases sharply in the basal part of the stem (Hakkila 1966).

Despite several studies on wood density, the vertical dependency has seldom been modelled (Björklund and Ferm 1984, Varjo 1991). To compare our result with other studies is not appropriate because, for instance, Bjöklund and Ferm (1984) concentrated on small-sized downy birch and grey alder, and Varjo (1991) constructed a model for pine wood density that included both a radial and vertical dependence.

The model proved to be a suitable method for determining the average wood density of the stem, because both the average vertical dependence of wood density (fixed effect) and its variation between trees (random effect) could be utilized. This method offered one way to improve the accuracy of estimates and decrease the risk of systemic errors when only a few sample disks per tree have been taken. The prediction error for the estimates of the average wood density for the stem was small when the model was used with one or two sample disks. When the wood density was determined by a (basal area weighted) mean of one or two sample disks, the overestimates were systematic for all three tree species, and especially for pine stems where the vertical dependence is strong. The traditional methods, based on the mean density of sample disks, would have produced a more significant error for larger-sized stems. For example, Hakkila (1979) reported clearly higher overestimates of the average wood density based on breast height measurements for birch and spruce stems.

Because the vertical dependence of the wood density varies among trees, the treewise prediction error can be decreased by increasing the number of sample disks. The error in the average wood density estimation also leads to an error, for example, in stem biomass estimates based on stem volume and the average wood density. This is most significant for tree species with a high vertical dependence of wood density.

There are some limitations to our material, and these must be kept in mind when generalizing the results. Only trees from final cuttings represented the total growing stock. The stems from the first and second commercial thinnings were concentrated on small diameter classes in the stand. Some differences in wood properties between suppressed and dominant trees have been reported. Hakkila (1979) found that stem wood density was on the average 1.5% higher, due to a slower growth rate, in the spruce, pine and birch material collected from thinning stands than in the material representing the total growing stock. We used the interaction of diameter and tree age as a variable to depict the growth rate, which presumably partly eliminated this difference between tree size classes. According to Kärkkäinen (1984), suppressed spruce stems have a lower wood density and dominant trees a higher density than would be expected on the basis of their growth rate. We used a constant coefficient for all tree classes, which may lead to over- or underestimation of the wood density depending on the tree class. Lindström (1996) showed that spruce wood density increases on moving from the pith outwards at a faster rate in suppressed trees than in dominant trees. This has a direct effect on vertical dependence, because vertical dependence is mainly due to the radial dependence of wood density. In any case, the growth rate was highly correlated with wood density in pine, spruce and birch stems in our study. Corresponding negative correlation has been detected in many conifer studies (Mergen et al. 1964, Hakkila 1979, Saranpää 1983, MacPeak et al. 1990, Mäkinen and Uusvaara 1992) and in birch studies (Hakkila 1979, Dunham et al. 1990)

The study material did not include any stems from young stands, and the minimum diameter of the sample trees was 6 cm. The vertical dependence of wood density in young pine and downy birch has been reported to be less than that in older trees (Hakkila 1966, Björklund and Ferm 1992). As the birch material consisted only of trees from the first commercial thinning, larger-sized birch stems were also lacking.

In our study the density estimates were based on knot-free sample disks. The wood density in knots and also around them is higher than in knot-free wood. Hakkila (1979) presented a knot correction of +1% for the dry weight of spruce, pine and birch. This also means that a correction of similar magnitude should be applied to the average wood density in order to obtain realistic values.

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