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SOME ECONOMIC ASPECTS OF GROWING FOREST STANDS

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TIIVISTELMÄ:
ERÄITÄ TALOUDELLISIA NÄKÖKOHTIA METSIKÖIDEN KASVATUKSESSA

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The soil value, calculated by application of the classic Faustmann formula, indicates the difference between the future proceeds and outlays in respect of a bare area which is reforested immediately. The proceeds correspond to the harvesting revenues. This paper demonstrates another means of arriving at the Faustmann formula. The difference between the current value growth and the rent of the growing stock is used in place of harvesting revenues. This method yields results which are exactly the same as those derived by the original Faustmann formula, but provides greater understanding of its applications.

1. INTRODUCTION

One of the most important tasks which confront foresters is that of determining the optimum rotation, growing density, and structure of the forest stand. This paper concentrates upon some economic aspects of the questions concerned. It is assumed that the aim in growing the forest stand is maximazation of the net present-value of future revenues from the stand. Since the proceeds and outlays occur during a long period, it is necessary to give them different weights at different points of time. This can be effected by introducing a guiding rate of interest, applied in calculation of discounting and compounding factors. By comparison of the net present-values, the profitability of different growing schedules can be evaluated.

Classic examples of present-value calculations are those made by application of the Faustmann formula, which indicates the net present-value of future revenues from bare land that is reforested immediately. This paper explores the Faustmann formula, with the aim of finding a method for determination of the optimum growing schedule for a forest stand with a guiding rate of interest given. It is assumed that the guiding rate of interest, timber prices, productivity of the soil, and the purpose for which the stand is grown remain unchanged in time.

2. AN INTERPRETATION OF THE FAUSTMANN FORMULA

The net present-value of the future revenues from a forest stand can be calculated by means of the formula:

$$(1)\quad \pi = \frac{\sum_{t=q+1}^{u} R_t r^{u-t} - \sum_{t=q+1}^{u} C_t r^{u-t} + L}{r^{u-q}}$$

where π = net present-value of the future revenues
R_t = revenue from harvest in year t
C_t = costs in year t
L = soil value
u = rotation length
q = present age of stand
$r = 1 + \frac{p}{100}$ (p = guiding rate of interest)

Formula (1) is a generalization of the »general present net worth formula» presented by Bentley and Teeguarden(1965, p. 86). Index q+1 is explained by the assumption that all revenues and costs occur at the end of each year.

When the present stumpage value of growing stock (V_q) and the soil value (L) are subtracted from the net present-value (π), the difference (D) expresses the loss in the net present-value of the future revenues, if the stand is clear-cut immediately, instead of its growth being continued for the next u—q years:

$$(2)\quad D = \pi - V_q - L$$

If the present age of the stand (q) is zero, then D and V_q are zero, π equals L, and L can be solved from formula (1):

$$(3)\quad L = \frac{\sum_{t=1}^{u} R_t r^{u-t} - \sum_{t=1}^{u} C_t r^{u-t}}{r^u - 1}$$

This is the familiar Faustmann formula in the form given by Bentley and Teeguarden (1965, p. 86).

Following this, another approach is made to calculating net present-values. It is suggested that the loss in the net present-value of future revenues, if the stand is clear-cut instead of its growth being continued for a further u—q years, can be calculated by the following formula:

$$(4)\quad D = \frac{\sum_{t=q+1}^{u} \left[Z_t - \frac{p}{100} V_{t-1} \right] r^{u-t} - \sum_{t=q+1}^{u} C_t r^{u-t} - L\, r^{u-q} + L}{r^{u-q}}$$

where V_t = stumpage value of growing stock in year t
Z_t = current value growth in year t

If the present age (q) equals zero, then D equals zero, and the soil value (L) can be derived from formula (4):

$$(5)\quad L = \frac{\sum_{t=1}^{u} \left[Z_t - \frac{p}{100} V_{t-1} \right] r^{u-t} - \sum_{t=1}^{u} C_t r^{u-t}}{r^u - 1}$$

Formula (5) differs from Faustmann formula (3) in the respect that the entity $Z_t - \frac{p}{100} V_{t-1}$ has been used instead of harvesting revenues. In this study, this entity is termed the »current gross soil rent». The current gross soil rent indicates the difference between the current value growth and the rent of growing stock. Figure 1 presents an example of the dependence of the current

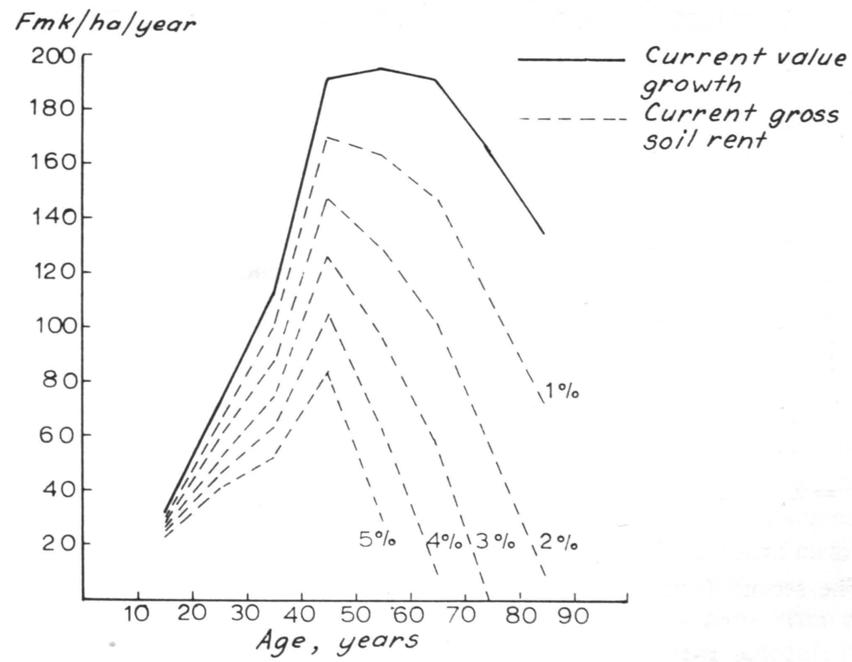

Figure 1. Current gross soil rent as a function of the rate of interest. Site MT.
Piirros 1. Maan juokseva bruttokorko korkokannan funktiona. Metsätyyppi MT.

gross soil rent on site, age, and guiding rate of interest. The figure is based upon data published by NYYSSÖNEN (1958).

The following derivation shows that formula (5) and Faustmann formula (3) yield results which are exactly the same.

The first term in the numerator of formula (5) can be rewritten:

$$\sum_{t=1}^{u}\left[Z_t - \frac{p}{100}\sum_{k=1}^{t-1}(Z_k - R_k)\right] r^{u-t}$$

where R_k = revenue from the intermediate cut in year k

Since r—1 equals $\frac{p}{100}$, the term can be derived further:

$$\sum_{t=1}^{u}\left[Z_t - \sum_{k=1}^{t-1} Z_k r + \sum_{k=1}^{t-1} Z_k\right] r^{u-t} + \sum_{t=1}^{u}\left[\sum_{k=1}^{t-1} R_k r - \sum_{k=1}^{t-1} R_k\right] r^{u-t}$$

The first term of this sum can be derived further:

$$\sum_{t=1}^{u}\left[\sum_{k=1}^{t} Z_k - \sum_{k=1}^{t-1} Z_k r\right] r^{u-t} = \sum_{t=1}^{u}\sum_{k=1}^{t} Z_k r^{u-t} - \sum_{t=1}^{u}\sum_{k=1}^{t} Z_k r^{u-t+1} =$$

$$\begin{array}{lllll}
Z_1 r^{u-1} & & & & \\
+ Z_1 r^{u-2} & + Z_2 r^{u-2} & & & \\
\vdots & & & & \\
+ Z_1 r & + Z_2 r & + \ldots & + Z_{u-1} r & \\
+ Z_1 & + Z_2 & + \ldots & + Z_{u-1} & + Z_u \\
- Z_1 r^{u-1} & & & & \\
- Z_1 r^{u-2} & + Z_2 r^{u-2} & & & \\
\vdots & & & & \\
- Z_1 r & - Z_2 r & - \ldots & - Z_{u-1} r &
\end{array}$$

$$= Z_1 + Z_2 + \ldots + Z_u = \sum_{t=1}^{u} Z_t$$

The second term can be developed similarly:

$$\sum_{t=1}^{u}\sum_{k=1}^{t-1} R_k r^{u-t+1} - \sum_{t=1}^{u}\sum_{k=1}^{t-1} R_k r^{u-t} =$$

$$\begin{array}{llllll}
R_1 r^{u-1} & & & & & \\
+ R_1 r^{u-2} & + R_2 r^{u-2} & & & & \\
\vdots & & & & & \\
+ R_1 r & + R_2 r & + \ldots & + R_{u-1} r & & \\
- R_1 r^{u-2} & & & & & \\
- R_1 r^{u-3} & - R_2 r^{u-3} & & & & \\
\vdots & & & & & \\
- R_1 r & - R_2 r & - \ldots & - R_{u-2} r & & \\
- R_1 & - R_2 & - \ldots & - R_{u-2} & - R_{u-1} &
\end{array}$$

$$= \sum_{t=1}^{u-1} R_t r^{u-t} - \sum_{t=1}^{u-1} R_t$$

The original sum can be rewritten:

$$\sum_{t=1}^{u} Z_t + \sum_{t=1}^{u-1} R_t r^{u-t} - \sum_{t=1}^{u-1} R_t = \left[\sum_{t=1}^{u} Z_t - \sum_{t=1}^{u-1} R_t\right] + \sum_{t=1}^{u-1} R_t r^{u-t}$$

The term in brackets indicates the value of the growing stock at the end of the rotation, and the whole sum can be rewritten:

$$R_u + \sum_{t=1}^{u-1} R_t r^{u-t} = \sum_{t=1}^{u} R_t r^{u-t}$$

It has thus been proven that the new formula (5) can be derived from Faustmann formula (3), and that both of the formulae give results which are exactly the same.

It is observable from formula (4) that the net present-value of the future revenues from the stand can be increased by raising the present-value of the current gross soil rents, and by cutting the present-value of the costs and soil. The present-value of the current gross soil rents can be changed either by alteration of the structure and density of the growing stock or by changing the rotation. These changes can be effected by cuts and other silvicultural measures, and consequently they also exercise a marked influence on the costs. As the soil value expresses the net present-value of the future soil rents after clear-cutting, it can be said in conclusion that the optimum growing schedule is the one that maximizes the net present-value of the future soil rents. The soil rent expresses the difference between the gross soil rent and the costs.

3. OPTIMUM STRUCTURE AND DENSITY OF THE STAND

It was assumed that the aim in growing the forest stand was that of maximizing the net present-value of the future revenues (p. 225). For fulfilment of this aim, the structure and density of the growing stock need to be so regulated that the net present-value of the current soil rents attains the maximum level. If the problem is simplified, and it is assumed that the history of the stand does not exercise any influence upon the present growth rate, this can be done by maximizing d_t in the following function:

$$(6) \qquad d_t = Z_t - \frac{p}{100} V_{t-1} - C_t$$

where d_t = current soil rent

If it is assumed that the annual cost (C_t) is constant, the problem is that of finding the structure and density of the growing stock in year t—1 which yields the maximum soil rent in year t. This is exactly what has been done in determination of the optimum density of selection forests (DUERR and BOND 1952).

Formula (6) indicates that valuable growing stock can be motivated only by high value growth. If the stumpage value of growing stock can be reduced without serious reduction of the value growth, this is clearly a profitable undertaking. Formula (6) also indicates that the optimum growing density may differ quite appreciably from that obtained by marginal analysis (DUERR and BOND 1952) in which it is assumed that the logging costs are independent of the thinning intervals and the amount of thinning removal. In a realistic situation, all the variables in formula (6), including the costs, have to be taken into account in determination of the optimum structure and density of the stand.

4. OPTIMUM ROTATION

Discussion here is concerned first with the difference between the rotations based upon the fully regulated forest model, and upon the Faustmann formula.

As the fully regulated forest model is employed in seeking the optimum rotation, the purpose is that of finding the rotation which maximizes the average soil rent ($\overline{d}$) derived by the formula:

$$(7) \qquad \overline{d} = \frac{1}{u} \sum_{t=1}^{u} (Z_t - \frac{p}{100} V_t - C_t) ,$$

in which the symbols correspond to those used in formula (4). Figure 2 illustrates the development of the average soil rent derived by formula (7). As the current soil rent (in parentheses in formula 7) falls below the average soil rent, the stand has reached the optimum rotation.

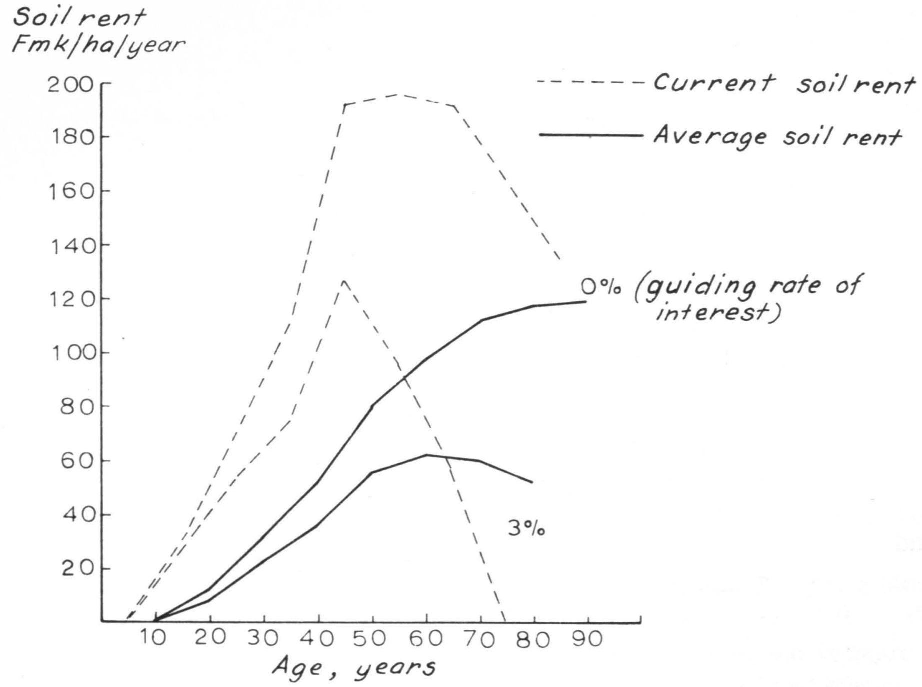

Figure 2. Graphical determination of the optimum rotation for the fully regulated forest model. Site MT. Regeneration cost 60 Fmk/ha.
Piirros 2. Optimikiertoajan graafinen määrittäminen normaalimetsälle. Metsätyyppi MT. Uudistuskustannus 60 mk/ha.

When the Faustmann formula is applied for discovery of the optimum rotation, the aim is that of finding the rotation which maximizes the net present-value of all future soil rents. In this case, the average soil rent equals $\frac{p}{100}$ L. In a way similar to the fully regulated forest model, the stand has reached the optimum rotation, as the current soil rent falls below the average soil rent.

In view of the difference in calculating the average soil rent, it is easily understandable that the previous two approaches for estimation of the rotation may lead to divergent results. This phenomenon has been noted by NYYSSÖNEN (1958, p. 21) and DUERR (1960, pp. 134—135).

As the guiding rate of interest is zero, the two methods give exactly the same rotation as that termed the rotation of the maximum forest rent (NYYSSÖNEN

1958, p. 20). The soil value is then infinitely high, and the current soil rent equals the current forest rent or the current net value growth (figure 2).

As the current soil rent falls below zero, the stand has reached the rotation which maximizes the net present-value of the soil rents from one rotation. This rotation, again, is longer than that obtained by application of the Faustmann formula (figure 2) (DUERR 1960, p. 133).

The following discussion is concerned with the loss in the net present-value of the future revenues if the stand is clear-cut immediately instead of being allowed to grow for one more year. A one-year period has been selected for the purpose of simplification. The question of growing the stand for only one more year is relevant only for stands with a current soil rent already declining (figures 1 and 2).

If q equals u—1 in formula (4), then D expresses the loss in the net present-value of the future revenues if the stand is clear-cut immediately instead of being allowed to grow for one more year. The stand has reached financial maturity when D falls below zero, and will not, moreover, exceed zero in the future. Then:

$$(8) \qquad 0 = Z_t - \frac{p}{100} V_{t-1} - C_t - \frac{p}{100} L$$

and

$$(9) \qquad p = \frac{Z_t - C_t}{V_{t-1} + L} 100$$

Formula (9) expresses the internal rate of return (p) of growing any stand of t—1 years for one more year. In forestry literature, this internal rate of return is termed the indicating percentage (in German: Weiserprozent) and a comprehensive treatment of this topic has been published by ENDRES (1923, pp. 229—254). Other formulae have also been developed for calculation of the indicating percentage, especially when the period of calculation exceeds one year (ENDRES 1923, pp. 235—247). As the indicating percentage equals the guiding rate of interest, the stand has reached financial maturity.

If the concept of internal rate of return is applied in the orthodox manner in accordance with the principles of the Faustmann formula, the soil value (L) should always represent the maximum soil value from formula (5). Furthermore, the structure and density of the growing stock should represent the optimum conditions in calculation of the soil values. An attempt to use the optimum densities of growing stock in calculation of the soil values has already been made by CHAPPELLE and NELSON (1964).

Formula (7) demonstrates that when the structure and density of the growing stock are at an optimum, the current soil rent (d_t) remains positive for as long as is possible. Similarly, the indicating percentage exceeds the guiding rate of interest for as long as possible if the structure and density are at the optimum, and the growth of the stand can be continued. Of course, this implies that the same guiding rate of interest is used in determination of both the optimum structure and density, and the financial maturity.

5. SUMMARY AND CONCLUSIONS

The aim of this paper is that of studying the optimum growing schedules of forest stands, with the classic Faustmann formula as starting point. The study is theoretical in nature, except in respect of the measured data given in figures 1 and 2.

The study shows that the net present-value of the future revenues from a forest stand can be calculated, not only by means of the harvesting revenues, but also by a more theoretical concept, here termed the current gross soil rent. The current gross soil rent represents the difference between the current value growth and the rent of the growing stock.

By use of the concepts described here, it is theoretically possible to find the growing schedule for the stand which maximizes the net present-value of the stand. To make the formulae simpler, a one-year period has been adopted for discussion of the concepts involved in determination of the optimum structure and density of the growing stock, and the financial maturity. However, these concepts can be extended to cover periods of any length.

The method for determination of the optimum growing schedule for a forest stand can be summarized as follows:

Thin the stand as the internal rate of return on the marginal increase in »timber capital» falls below the guiding rate of interest. Clear-cut and regenerate the stand as the internal rate of return on the sum of the »timber and soil capital» falls below the guiding rate of interest.

TIIVISTELMÄ:

ERÄITÄ TALOUDELLISIA NÄKÖKOHTIA METSIKÖIDEN KASVATUKSESSA

Kiertoajan, kasvatustiheyden ja kasvatettavan puuston rakenteen optimointi on eräs metsäammattimiehen keskeisimmistä tehtävistä. Tässä työssä pidetään metsiköstä saatavien nettotulojen nykyarvoa kasvatustavan edullisuuden kriteerinä. Tutkimus osoittaa, että metsiköstä saatavien bruttotulojen nykyarvo voidaan laskea diskonttaamalla nykyhetkeen vuotuisten arvokasvujen ja puuston korkojen erotus ja lisäämällä tähän puuston hakkuuarvo laskentahetkellä. Tällä menetelmällä saadaan täsmälleen samoja tuloksia kuin diskonttaamalla kantorahatulot nykyhetkeen. Käytetyn menetelmän suurin etu on siinä, että se tuo esille metsikön nykyarvoon vaikuttavat perustekijät — arvokasvun ja puuston koron. Hakkuut voidaan täten nähdä vain kertyneen arvokasvun realisointina, ja hakkuiden ajoitus on tehtävä siten, että nykyhetkeen diskontattu arvokasvun ja puuston koron sekä kustannusten erotus maksimoituu.

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