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Prediction of tree diameter and height in a Scots pine stand as a function of the spatial pattern of trees

Timo Pukkala

TIIVISTELMÄ: PUUN LÄPIMITAN JA PITUUDEN ENNUSTAMINEN TILAJÄRJESTYKSEN AVULLA MÄNNIKÖSSÄ

Pukkala, T. 1989. Prediction of tree diameter and height in a Scots pine stand as a function of the spatial pattern of trees. Tiivistelmä: Puun läpimitan ja pituuden ennustaminen tilajärjestyksen avulla männikössä. Silva Fennica 23(2): 83–99.

The study presents two methods of predicting tree dimensions in a Scots pine stand if only the locations of trees are known. The first method predicts the tree diameter from the spatial location of neighbors. In the second method the diameter distribution of a subarea is estimated from the local stand density. This distribution is then sampled to obtain diameters. In both methods the tree height is predicted with a spatial model on the basis of diameters and locations of trees. The main purpose of the presented models is to generate realistic stands for simulation studies.

Tutkimuksessa tarkastellaan kahta tapaa ennustaa männikön puiden läpimitta ja pituus, kun ainoastaan puiden tilajärjestys tunnetaan. Ensimmäisessä menetelmässä ennustetaan suoraan puun läpimittaa naapuripuiden lukumäärän ja aseman perusteella. Toinen menetelmä perustuu pienehkön osa-alueen läpimittajakauman ennustamiseen kyseisen alueen runkoluvun perusteella. Puun pituus lasketaan kummassakin menetelmässä spatiaalisella pituusmallilla puiden läpimitan ja keskinäisen aseman perusteella. Esitettyjen mallien päätarkoitus on kyetä generoimaan realistisia mallimetsiköitä simulointitutkimuksiin.

Keywords: simulation studies, diameter distribution, spatial distribution, competition, tree models.
ODC 521 + 522 + 53

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Symbols

A	Grouping index of Hopkins
AS	Sum of horizontal angles from the object tree to the stems of neighbors nearer than 6 m, calculated 1.3 m above the ground level (radian)
d	Breast height diameter (cm)
$d_c(g)$	Mean diameter within a circle with radius 6 m, weighted by tree basal area (cm)
$d_c(n)$	Unweighted mean diameter within a circle with radius of 6 m (cm)
d_{max}	Maximum diameter (cm)
d_{min}	Minimum diameter (cm)
$d_s(g)$	Mean diameter of the stand weighted by basal area (cm)
D	Mean distance of neighbors nearer than 6 m (m)
G_c	Basal area of trees within 6 m from the object tree, object tree included (m²/ha)
G_s	Stand basal area (m²/ha)
h	Tree height (m)
I	Grouping index of Fisher
ID	Sum of inverted distances from the object tree to neighbors which are nearer than 6 m (1/m)
N_c	Number of trees within 6 m from the object tree, object tree included (trees/ha)
N_s	Mean stocking of the stand (trees/ha)
N_{a-b}	Number of trees whose distance from the object tree is between a m and b m, a included (trees/ha)
$N_{<a}$	Number of trees whose distance is less than a m from the object tree, object tree included (trees/ha)
R	Grouping index of Clark and Evans
s_d	Standard deviation of diameter (cm)
S	Distance from the object tree to the mean x- and y- coordinate of neighbors that are nearer than 6 m (m)

1. Introduction

In an even-aged stand composed of one tree species the stand structure is defined by the size distribution and the spatial distribution of trees. The size distribution is usually described in terms of diameter distribution. The description and estimation of diameter distribution (Päivinen 1980, Kilkki and Päivinen 1986) and spatial distribution (Matérn 1960, Tomppo 1986) and how they affect growth (Mielikäinen 1978, Nyyssönen and Mielikäinen 1978, Pukkala and Kolström 1987, Pukkala 1988, 1989) are basic problems of forest mensuration and forest yield studies.

The dependence of growth on diameter distribution is described by non-spatial growth models and the joint effect of the size and spatial distribution by spatial growth models. Although there are many spatial growth models (Ek and Monserud 1974, Adlard 1974, Eriksson 1976, Saramäki 1977, Mielikäinen 1978, 1980, Pukkala

1989), the interdependence between diameter distribution and spatial distribution has only seldom been the object of research (Väliaho 1971, Tomppo 1986). It is reasonable to assume, however, that this interdependence is strong and can be used to characterize the stand (Klier 1969). In a fully stocked stand the average size of trees must be smaller in a dense spot than in a sparse one if the carrying capacity of the site is uniform.

The relationship between spatial distribution and diameter distribution is important in both directions: the prediction of the spatial distribution from the size distribution and from other stand characteristics is relevant e.g. in such a forest inventory which measures the diameter distribution only (Tomppo 1986). The opposite direction, i.e. the estimation of the diameter distribution from the spatial pattern of tree locations, is important in simulation studies that examine the dependence of stand productivity on the spatial properties of the stand (Lundell 1973, Eriksson 1977, Pukkala 1988, 1989); the diameter distribution must be related to the spatial one in a realistic manner. Otherwise the growth models used in the simulation may give unreliable estimates for some subareas or the simulated stands may be unrealistic.

It is important that the relation between the tree size and number of trees is realistic everywhere in the stand, not only at the level of the whole stand. It is probable that not only the diameter but also the shape of the stem depends on stand density in the vicinity of the tree. It is therefore necessary to study the effect of the variation in stand density on tree height as well (Perry 1985).

In principle, it is possible to generate a sensible joint distribution for the tree location and tree size by using models on stand dynamics only. The initial stand is first generated by a spatial birth model (Kellomäki et al. 1987), after which tree dimensions are increased by spatial growth models and death predicted by spatial mortality models. Such models are not, however, available at the moment, and their construction is not easy. In addition, the spatial questions often arise at a later stage of stand development, e.g. when the changing of the stand structure becomes topical through thinnings (Eriksson 1977, Bucht 1981).

This study examined the effect of the pattern of the tree location on tree size in Scots pine stands. The aim was to prepare spatial models for predicting diameter and height. A method for generating model stands with a given spatial pattern is presented as an example of the use of the models.

The study includes references to such concepts as stand density, local density and local diameter distribution. In this paper, stand density is the number of trees per unit area. Local density is the number of trees (trees/ha) in a small subarea of the stand and local diameter distribution the frequency distribution of diameter in a small subarea.

2. Material

The dependence of the tree size on the spatial distribution of trees was studied within one thoroughly measured Scots pine stand and with 50 sample plots measured in different stands. Within the individual stand the aim was to examine the nature of the dependencies between the spatial pattern and tree size, and to find out the best predictors for the spatial diameter and height model. A 170 m x 155 m area (2.635 ha) in a 70-year-old Scots pine stand (63°40'N, 31°5'E) was measured tree by tree

by species, diameter and coordinates (the material is the same as in Pukkala (1989) except that an additional 5-m-wide buffer zone was measured around the area). The tree height was measured from all trees further than 5 m from the plot edge. The measured area included 2199 trees (834 trees/ha). The mean diameter of the stand was about 21 cm, mean height 18 m, stand basal area 18 m²/ha and stand volume 140 m³/ha. Only trees whose dbh was at least 3 cm were measured in the field.

The fertility of the site is reasonably uniform over the whole area. The degree and type of aggregation of trees varies considerably (see Pukkala 1989), as well as the tree size (Fig. 1). These properties render the area suitable for the present study.

In the second phase of the study the aim was to prepare more general spatial models for Scots pine for estimating tree diameter and height. The material consisted of 50 sample plots measured in pure Scots pine stands in North Karelia (63°40'N, 31°5'E), Finland. The size of the sample plot varied between 800 m² and 3000 m² depending on the stand density and size of trees. Each tree was measured by coordinates and diameter and at least 20 trees/plot by height. Only trees with dbh at least 3 cm were included in the computations.

The plots varied in the stage of development from rather young stands still in the sapling phase to old stands where the mean diameter was over 35 cm (Appendix 1). The stands were even-aged and the site was uniform within each plot. The stand density and the spatial distribution of trees var-
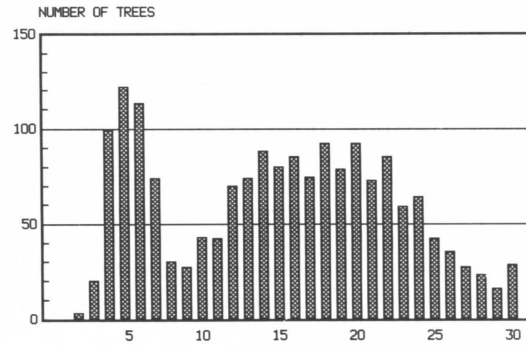

Fig. 1. Diameter distribution of the sample plot used in the within-stand analyses.

ied considerably from plot to plot. All the stands were naturally regenerated. For this reason, there were no strictly regular stands in the study material.

The variation in the size of trees and in their aggregation was partly due to natural reasons and partly due to human intervention. Some of the sample plots were situated beside a road or a telecommunication line, some even crossed narrow treeless corridors. The stands had been managed normally, which means that thinnings had been carried out according to a normal timetable. In all the thinned plots at least five years had elapsed since the last thinning treatment. The irregularities due to other reasons than thinning usually had a still older origin.

3. Methods

31. Estimation of breast height diameter

The models for tree diameter and height were computed in two stages. First, a diameter model was calculated with the help of the spatial pattern of trees. Secondly, a height model was constructed on the basis of tree locations and diameters. Two different approaches were utilized to predict the

tree diameter. In the first method the diameter was predicted directly from the position of neighbors in the vicinity of the object tree (referred to as Method 1). With this method the individual diameters are simulated as the sum of the model prediction and a stochastic component corresponding to the residual variation of the model.

The location of neighbors was described by their average distance, density (trees/ha) at different distances and their directional distribution. Only neighbors nearer than 6 m were used for predicting the diameter, because it has been found that trees further than that affect very little diameter growth (Eriksson 1977, Bucht 1981, Pukkala and Kolström 1987, Pukkala 1988, 1989). The predictors of the diameter model were computed for each tree whose distance from the nearest plot edge was at least 6 m.

32. Estimation of local diameter distribution

The second method of predicting the tree size was to estimate the local diameter distribution in a subarea around the object tree (object tree included) and sample that distribution to obtain diameters (Method 2). The subarea was again a circle with a radius of 6 m, the object tree locating in the center. The beta function was selected as a theoretical distribution because of its flexibility which allows many types of distributions to be described. For each tree further than 6 m from the plot edge the following stand characteristics needed for predicting the beta distribution were computed: number of trees per hectare, minimum, mean, maximum and standard deviation of diameter. This data were used for constructing models that facilitated the prediction of the mean, range and variance of the local diameter distribution from the local stand density.

33. Measures of aggregation

When predicting the tree diameter and height with spatial models, some stand characteristics besides tree coordinates are usually known. In the present study it was assumed that the mean stocking (trees/ha) and the stand basal area are known and they can thus be used for predicting the tree size. Additional predictors, which describe the aggregation pattern and grouping of

trees at the stand level, can be computed from the coordinates of trees. In this study the following three grouping indices were calculated for each of the 50 sample plots and used as potential predictors of the diameter, diameter distribution and height.

The index R of Clark and Evans (1954) is

$$R = v^2 \sqrt{\mu} \qquad (1)$$

where v is the observed mean distance between neighboring trees, and μ is the stand density (trees/m²).

The index A of Hopkins (1954) is defined by

$$A = a_i^2 / b_j^2 \qquad (2)$$

where a_i is a random point-to-tree distance and b_j random tree-to-neighbor distance. An equal number of each kind of distances is supposed.

The grouping index of Fisher (1922) is given by

$$I = s_n^2 / m \qquad (3)$$

where s_n^2 is the variance of the stem number in a subarea of certain size and m the average number of trees in the subarea. In this study the subarea was a square with an area equal to the mean growing space of one tree (plot area divided by number of trees). The squares were laid over the plot in a systematic manner so that the edges of the square were parallel to the plot edges.

In a Poisson forest the expected value of all the three indices is one. Also many other distributions than Poisson produce values near to one. In a completely regular stand (square lattice) the index R equals two and decreases down to zero with increasing aggregation (Leps and Kindlmann 1987). A and I greater than one indicate an aggregated pattern and values less than one some degree of regularity (I also depends on the size of the subarea). Each of the indices depends on the spatial pattern in a different way so that it can be supposed that they describe different aspects of the spatial distribution of trees (see Appendix 1).

4. Results

41. Within-stand models

Diameter model

The mapped area of 2.635 ha in the 70-year-old pine stand provided a large material to study the nature of the joint distribution of the tree size and tree location. In this stand the density around dominant trees is clearly different from that around the dominated trees (Fig. 2). The smallest trees usually occur in the densest spots of the stand and the biggest ones in the sparse areas. The number of trees per unit area increases towards small trees and decreases towards big trees. For dominant and average trees there is a dense zone at a distance of 2...4 meters from the tree after which the stand density remains constant or decreases a little. Inside this zone there is a sparsely populated area, which presumably is a consequence of competition. The fact that the dominated smallest trees can not suppress their neighbors to death is seen as a tendency of small trees gathering neighbors in their surroundings.
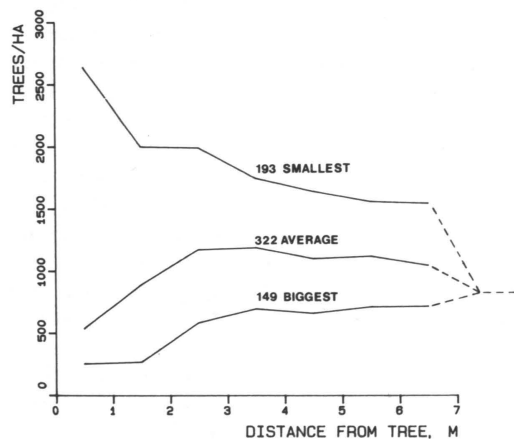
The dependencies depicted in Fig. 2 can be used as a basis for predicting the tree diameter. If the number of trees per hectare increases towards a tree, it is probably a small individual. If the stocking is very low in the immediate vicinity of the tree and then increases, the tree tends to be a dominating individual.

The first model for predicting diameter relies on this rationale; the stand densities (trees/ha) at different 1-m-wide zones around the object tree were used as potential predictors of diameter in addition to parameters that described the average distance and directional distribution of neighbors. The regression analyses resulted in the following spatial diameter model:

	Coefficient	t-value
$d^{0.4} =$	3.603	11.6
	$+ 0.1024\ D$	3.0
	$- 0.04500\ S$	2.5
	$- 0.4893\ \ln(ID+0.001)$	14.3
	$+ 0.00009885\ N_{3-4}$	4.3
	$- 0.08962\ N_{<2}$	2.6 (4)

$$R^2 = 0.316 \qquad s_e = 0.518$$

The strongest predictor of diameter is the sum of inverted distances from the object tree to neighbors nearer than 6 m (ID); it combines the effect of the number and distance of neighbors in a realistic manner. According to the model, diameter is the greater the smaller the number of neighbors within 2 m. The positive regression coefficient of N_{3-4} is a consequence of the dense zone at a distance of 3 – 4 m from big and average trees (see Fig. 2). This is probably the distance where the competitive effect of a tree sharply decreases and can thus be occupied by competitors.

Equation (4) explains only 32 % of the variation in $d^{0.4}$. The rest of the variation could not be accounted for by the variation in the number density of the stand. When predicting the diameter distribution by Equation (4) the greatest share of variation

Fig. 2. Stand density (trees/ha) in one-meter-wide zones at different distances from biggest trees, trees of average size and small trees.

is in the stochastic residual component. Its distribution is therefore of particular importance. The variance of the residual was found to be most constant and its distribution nearest to normal when the predicted variable was the diameter raised to power 0.3...0.5, which means that the local frequency distribution of diameter is between normal and log-normal.

Models for local diameter distribution

The models for estimating the parameters of the diameter distribution in a 113 m^2 subarea (a circle with radius 6 m) are as follows:

	Coefficient	t-value
$\ln(d_c(n)) =$	5.732	97.9
	$- 0.4460\ \ln(N_c)$	52.8 (5)

$$R^2 = 0.607 \qquad s_e = 0.194$$

	Coefficient	t-value
$\ln(d_{min}) =$	0.7216	2.8
	$- 0.2084\ \ln(N_c)$	8.8
	$+ 0.9252\ \ln(d_c(n))$	22.4 (6)

$$R^2 = 0.555 \qquad s_e = 0.340$$

	Coefficient	t-value
$\ln(d_{max}) =$	0.9492	9.0
	$+ 0.09202\ \ln(N_c)$	9.4
	$- 0.1179\ \ln(d_{min})$	12.3
	$+ 0.6760\ \ln(d_n(n))$	35.7 (7)

$$R^2 = 0.500 \qquad s_e = 0.138$$

	Coefficient	t-value
$\ln(s_d) =$	0.2370	5.6
	$- 0.1723\ \ln(N_c)$	29.5
	$+ 0.9480\ \ln(d_{max}-d_{min})$	110.4 (8)

$$R^2 = 0.871 \qquad s_e = 0.131$$

The models were estimated in such an order as gave the best average degree of determination; because the mean diameter correlated best with the stand density, it was estimated first and so on. The level of significance is high for all the models,
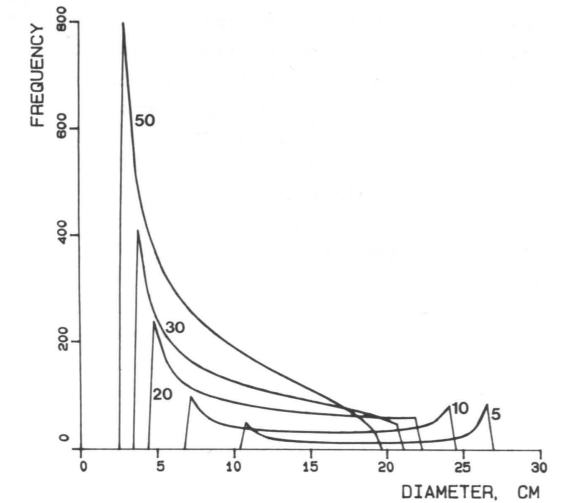

Fig. 3. Estimated diameter distribution in a circle whose radius is 6 m. The figures beside the curves show the number of trees within the circle. Numbers 5, 10, 20, 30 and 50 correspond to stand densities 442, 884, 1768, 2653 and 4421 trees/ha, respectively.

because of a great number of observations, although the degree of determination is not particularly high.

By using Equations (5)...(8) the parameters of the beta distribution can be calculated by the method of Loetsch et al. (1973). According to the models the diameter distribution shifts towards smaller diameters as the stand density increases (Fig. 3). With a low local density the distribution is bimodal: the trees within a subarea are either dominant trees or suppressed small individuals. By increasing the stand density the distribution approaches log-normal, i.e the additional trees are small individuals. The number of big trees does not increase with increasing stand density. These results are fairly logical in the light of the diameter distribution of the whole studied area (Fig. 1).

Validity of the models

The two methods were tested within two 50 m x 40 m subareas selected from different parts of the studied stand. By using the coordinates of the trees only, the diameters

of trees further than 6 m from the plot edge were generated by both methods presented above. In the first method a normally distributed stochastic term was added to the prediction to simulate the residual variation of the model. In the second method the local diameter distribution was predicted, each tree in turn being in the center of a circle of 311 m² (radius 6 m). One diameter was then randomly sampled from this distribution to obtain a diameter for the central tree. The term $s_e^2/2$ was always added to the expected value of Equations (5)...(8) due to the logarithmic transformation of the predicted variable.

The resulting spatial distributions of different diameters have the same appearance and outlook as in the real stand in both subareas and for both methods (Fig. 4). It is difficult to say, by inspecting the crown maps of the stands, which of them is real and which is simulated. Because both of the applied ways to obtain diameters are highly stochastic, the stands should not be com-

pared at the tree level but at the level of small subareas only.

The diameter distribution of the generated stands is always significantly different (according to χ^2-test) from the real distribution (Fig. 5). However, the mean diameter and the total basal area are fairly near to their correct values. It seems that both methods have difficulties in producing similar bimodal distributions as in the real stand, which is like a combination of two normal distributions. Method 1, which adds stochastic variation to the predicted diameter, can not at all produce bimodal local distribution, which according to Figs. 1 and 3 is a serious shortcoming in this particular stand. At the stand level, however, the distribution can be bimodal also with Method 1 provided that the dense and sparse subareas are large enough. Method 1 has also a tendency to increase the range of variation in diameter because the normal distribution of the stochastic term has no lower or upper limit.

Method 2, which is based on the local diameter distribution as described by the beta function, tends to cut away the smallest and biggest diameters if Equations (5)...(7) are used in a deterministic manner. It has no difficulties in producing bimodal distributions, although in the studied subareas they differ considerably from the real ones.

Height model

If diameter is predicted before height, as assumed in the present study, tree diameters can be used as predictors of the spatial height model, in addition to the tree coordinates. The model for predicting tree height in the studied stand is as follows:

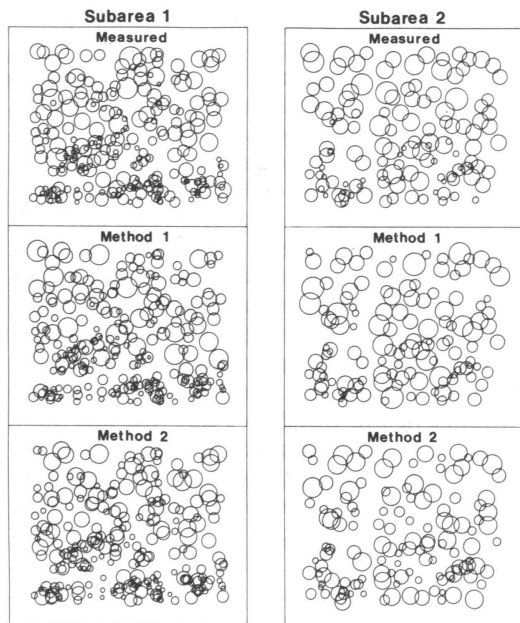|	Coefficient	t-value
$\ln(h-1.3) =$	4.097	114.6
	$-$ 0.4048 $d/d_c(g)$	16.2
	$+$ 0.1787 AS	19.2
	$-$ 22.92 $(1/(d+4))$	78.0 (9)
$R^2 = 0.929$		$s_e = 0.181$

Fig. 4. Real and estimated sizes of trees in two subareas of the studied stand. The radius of the circle indicates the greatest diameter of the crown and is linearly dependent on stem diameter (Crown width (m) = 0.5182+0.5183 d (cm)).
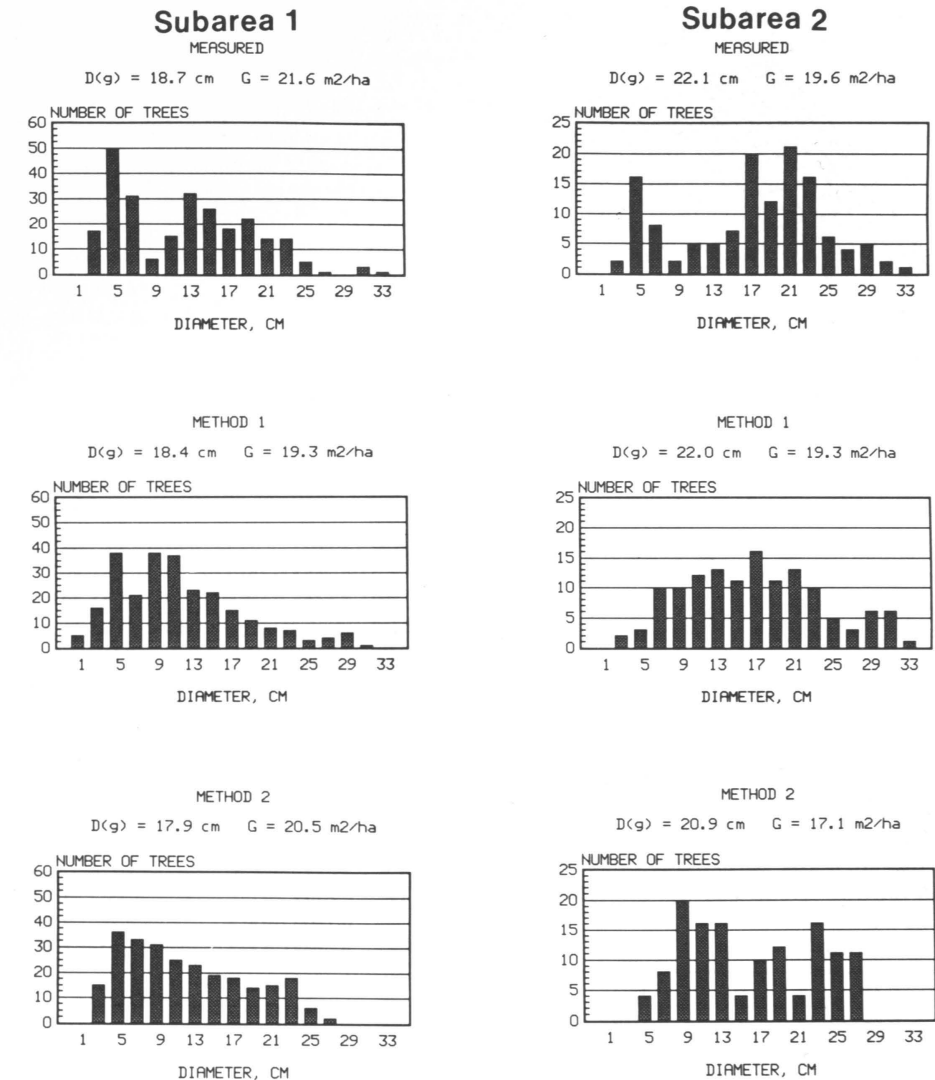
Fig. 5. Diameter distribution, mean diameter (weighted by tree basal area) and basal area of stands shown in Fig. 4.

The most important predictor is of course diameter ($1/(d+4)$), but the dependence of height on diameter is modified by the relative size of the tree with respect to its neighbors and the amount of competition. According to Equation (9), increasing relative size decreases and increasing competition increases height. Trees which have much competition are thus slenderer than solitary trees (Fig. 6).

42. General spatial models for diameter and height

Spatial diameter model

In this chapter it is assumed that the stand basal area and the mean stocking (trees/ha) are known besides tree coordinates when predicting diameter and height. Because there are now several stands in the

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Fig. 6. Dependence of the height of a tree whose diameter equals the local mean diameter on diameter and angle sum.

analyses, variables that describe the spatial pattern of the stand can be used in addition to the local stand density to predict tree dimensions.

The model that predicts the diameter directly has now the following form when the stand parameters are in the analysis:

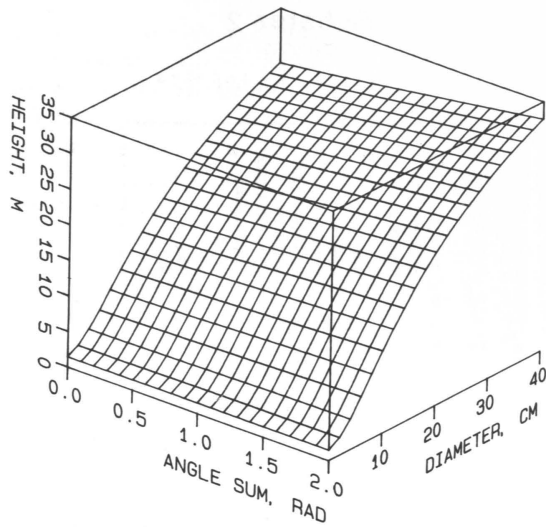|	Coefficient	t-value
$d^{0.5} =$	9.429	18.4
	$- 0.3835 \ln(N_s)$	10.4
	$+ 0.7399 \ln(G_s)$	15.0
	$- 0.1633 \ln(ID + 0.001)$	5.5
	$+ 0.0002134 \, N_{<4}$	6.6
	$- 0.2116 \ln(N_{<2})$	6.5
	$- 0.5310 \ln(N_{<4})$	9.1 (10)

$R^2 = 0.486$ $s_e = 0.734$

Logically, increasing the stand density decreases the predicted diameter and increasing the stand basal area increases it. Parameters R, A and I that describe the degree of aggregation were not significant predictors. Predictors calculated from the coordinates of the neighbors within 6 m from the object tree are partly the same as in the within-stand model (Eqn 4). The regression coefficients indicate that the

general features depicted in Fig. 2 are also valid when several stands are concerned.

The degree of determination is higher than for the withinstand model (Eqn 4), because of additional variables and greater variation in observations, but more than 50 % of the variation in $d^{0.5}$ remained still unexplained.

Models for local diameter distribution

The models for predicting the parameters of the local diameter distribution, when some stand parameters are available, are as follows:

	Coefficient	t-value
$\ln(d_c(n)) =$	4.567	96.0
	$- 0.2331 \ln(N_c)$	29.4
	$- 0.2538 \ln(N_s)$	30.1
	$+ 0.4771 \ln(G_s)$	41.8
	$+ 0.1881 \ln(R)$	5.3
	$+ 0.03535 \, A$	7.8
	$- 0.1330 \ln(I)$	8.0 (11)

$R^2 = 0.852$ $s_e = 0.153$

	Coefficient	t-value
$\ln(d_{min}) =$	$- 0.8144$	4.1
	$- 0.1390 \ln(N_c)$	8.7
	$+ 0.02971 \, d_c(n)$	7.3
	$+ 0.8956 \ln(d_c(n))$	12.6
	$+ 0.0001465 \, N_s$	14.1
	$+ 0.7507 \, R$	12.9
	$+ 0.09104 \, A$	11.4
	$- 0.2297 \, I$	7.9 (12)

$R^2 = 0.777$ $s_e = 0.287$

	Coefficient	t-value
$\ln(d_{max}) =$	1.986	22.2
	$+ 0.06579 \ln(N_c)$	8.4
	$- 0.2043 \ln(d_{min})$	27.4
	$+ 0.8179 \ln(d_c(n))$	45.7
	$- 0.1334 \ln(N_s)$	15.9
	$+ 0.01204 \, G_s$	20.0
	$- 0.3560 \, R$	15.2
	$+ 0.1040 \ln(I)$	7.3 (13)

$R^2 = 0.821$ $s_e = 0.135$

	Coefficient	t-value
$\ln(s_d) =$	1.789	27.5
	$- 0.4222 \ln(N_c)$	47.9
	$+ 0.00008663 \, N_c$	16.1
	$- 0.1028 \ln(d_{min})$	18.3
	$+ 1.048 \ln(d_{max}-d_{min})$	123.2
	$- 0.06224 \ln((d_{max}-d_{min})^2)$	9.2
	$+ 0.1106 \ln(G_s)$	12.9
	$- 0.1412 \ln(R)$	8.5 (14)

$R^2 = 0.989$ $s_e = 0.107$

The degree of determination of these models as well as the level of significance is considerably higher than for the within-stand models. In this case also the grouping indices are significant predictors. Although the individual coefficients must be interpreted with caution, because the predictors correlate with each other, it seems that by increasing the grouping of trees the mean and minimum diameters of a subarea decrease and the maximum diameter increases, all of which are very logical results (Tomppo 1986). The same phenomena can be seen in Fig. 7 which shows the estimated diameter distribution for different subareas in a regular and irregular stand. According to Equations (11)...(14) the increase in the local stand density (trees/ha) is mainly due to the increase in the number of small trees (Fig. 7), as it was in the within-stand analysis (Fig. 3).

Validity of the models

To test the validity of the two methods to predict diameter, four plots, all of which were different from each other in some respects, were removed from the study material (plots 25, 33, 37, 48) and Equations (10)...(14) were recomputed without them. The test material was thus independent of the study material. The coefficients of these new equations were near to those of Equations (10)...(14) and are not presented here. The models were used for generating tree diameters for the four test plots in the same way as explained in the within-stand analysis except that the mean stocking, stand basal area and the grouping indices were used as additional predictors.

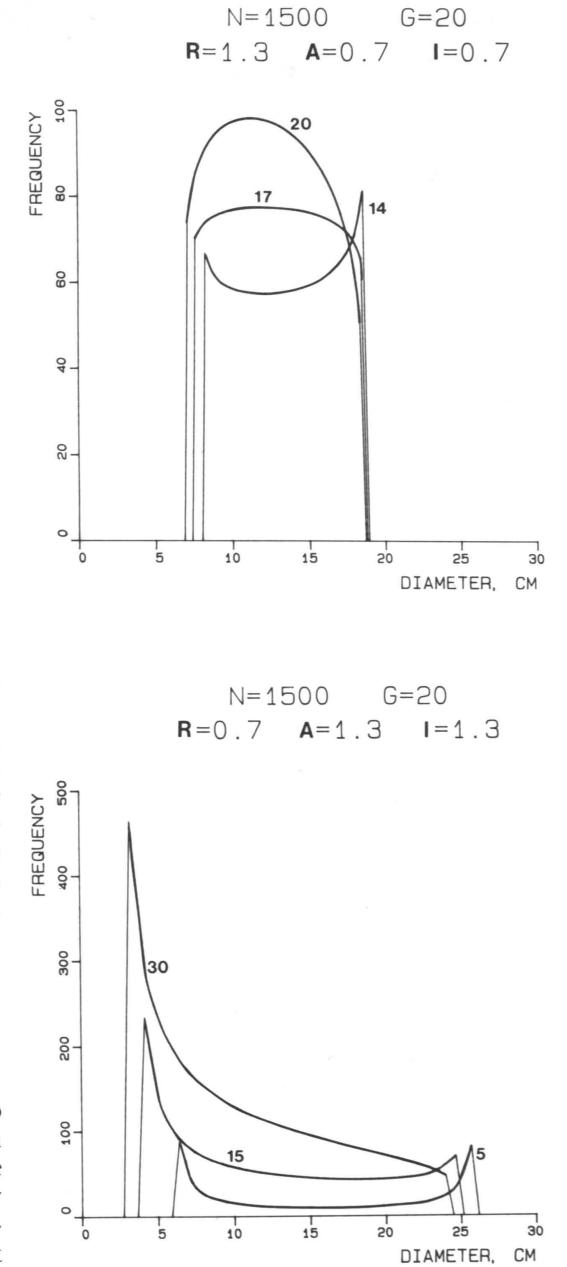Fig. 7. Estimated diameter distribution of a sub-area (circle with radius 6 m) of a stand whose mean stocking is 1500 trees/ha and basal area 20 m²/ha with regular (above) and aggregated (below) distribution of tree locations. The figures beside the curves show the number of trees in the sub-area. Numbers 5, 14, 15, 17, 20 and 30 correspond to 442, 1238, 1326, 1503, 1768 and 2625 trees/ha respectively.
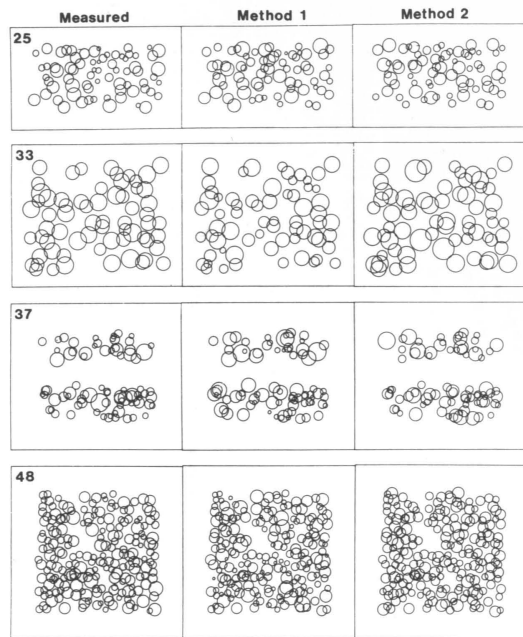
Fig. 8. Measured and estimated sizes of trees on four forest stand sample plots. The radius of the circle indicates the greatest diameter of the crown and is linearly dependent on stem diameter.

The generated diameters correspond again to the measured ones when viewed at the plot level (Fig. 8). The most striking discrepancy can be found in plot 33 where both methods generate more variation in diameter than can be found in the real stand. Especially Method 1, which predicts directly the diameter, has produced too many small trees.

The distributions of the measured diameters and the generated ones do not always differ significantly from each other (Fig. 9). The means of the simulated diameters and their total basal areas are nearer to the correct values in Method 2 (sampling from the predicted beta distribution). Also the diameter distribution of the plot matches better with the measured one in Method 2 than in Method 1. In some cases, however, Method 2 gives too narrow a range of variation (plots 25 and 48).

The methods were further compared by calculating the correlation coefficient of

Table 1. Correlation coefficient of measured or predicted diameter with the sum of inverted distances of neighbors nearer than 6 m (ID) in the 50 study plots.

	Measured diameter	Predicted diameter Method 1	Method 2
	correlation	coefficient	with ID
Average coefficient of 50 study plots	-0.27	-0.27	-0.18
Correlation over all plots	-0.56	-0.55	-0.55
	correlation	coefficient	with ln(ID)
Average coefficient of 50 study plots	-0.27	-0.28	-0.20
Correlation over all plots	-0.63	-0.63	-0.61

the measured or predicted diameter with the variable ID (the sum of inverted distances of neighbors nearer than 6 meters). The diameter was predicted and the value of ID calculated for all trees of the 50 study plots which were further than 6 meters from the plot edge (3521 trees). Method 1 gave almost exactly the same correlation between the predicted diameter and the sum of inverted distances of neighbors than was found in the measured stands (Table 1). This means that Method 1 produces similar joint distributions of tree diameter and the number and distance of neighbors as encountered in real stands. With Method 2 the correlations were somewhat poorer. In the present study material the correlation between diameter and variable ID was better than in Päivinen's (1987) material (-0.27 versus -0.20)

Height model

The height model with the mean stocking and stand basal area as additional predictors is as follows:
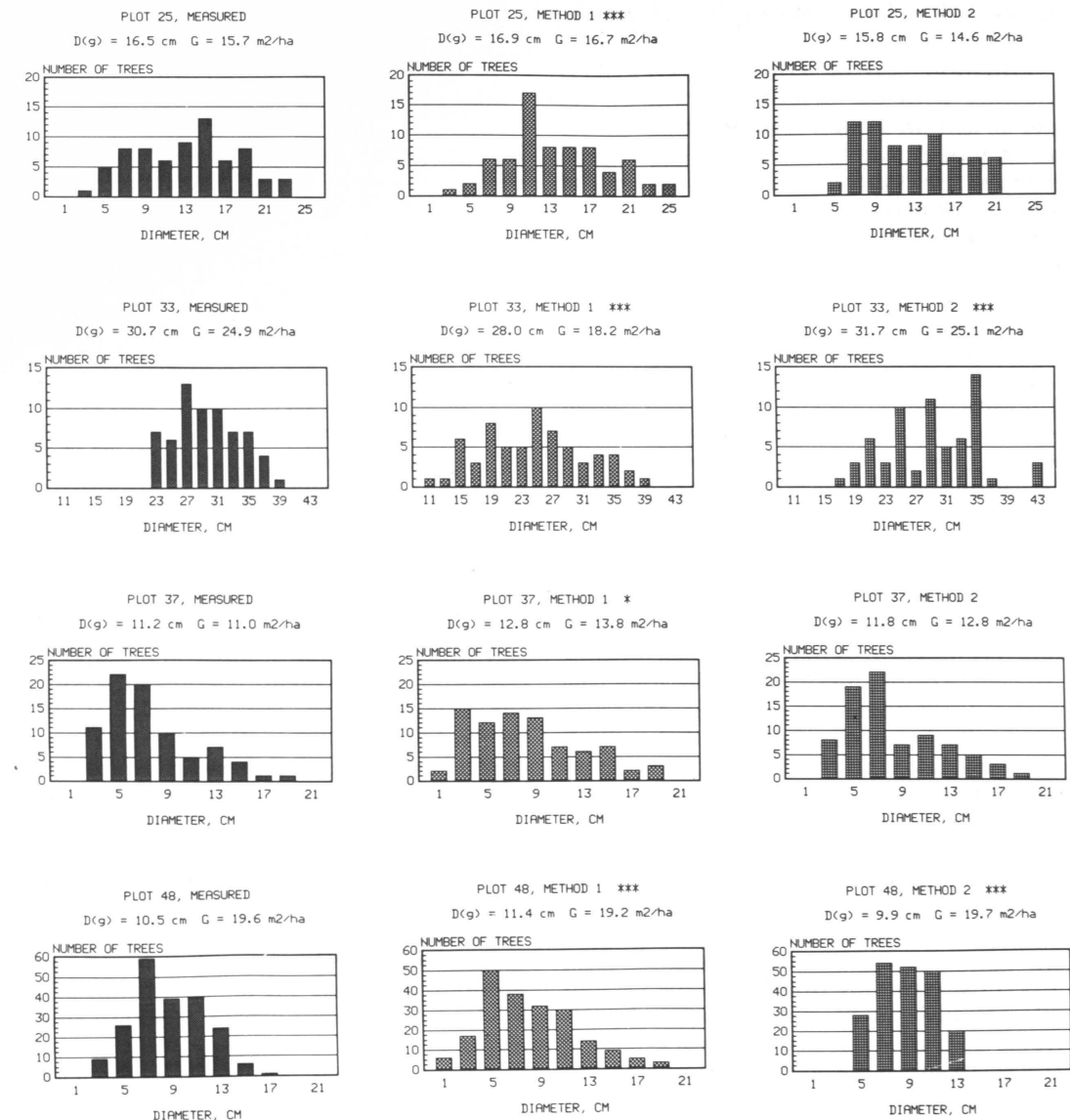
Fig. 9. Diameter distribution, mean diameter (weighted by tree basal area) and stand basal area of stands shown in Fig. 8.

	Coefficient	t-value
$\ln(h-1.3) =$	4.077	38.2
	$- 0.1018 \ln(N_s)$	7.0
	$+ 0.1823 \ln(G_s)$	6.6
	$- 0.3952\, d/d_c(g)$	10.1
	$+ 0.1491\, AS$	6.0
	$- 20.21\, (1/(d+4))$	38.5	(15)
	$R^2 = 0.861$	$s_e = 0.195$

The residual variation of this model is greater than that of Equation (9), presumably because the site fertility is not included in the model. The site could be taken into account by using stand age as a predictor. Another reason which decreases the degree of determination is that different stands have been thinned differently and the time since last thinning varies from plot to plot.

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43. Procedure for generating model stands

Equation (10) or Equations (11)...(14) together with the height model (Eqn 15) can be used in simulation studies for generating model stands in which the relation between the spatial variation in stocking and tree dimensions is realistic. The procedure to obtain tree dimensions has the following steps:

(1) Define the mean stocking and total basal area of the stand (N_s and G_s).

(2) Generate the tree coordinates as a realization of a suitable spatial process.

(3) Predict the diameters of the trees either by Eqn (10) or by Eqns (11)...(14). If Eqn (10) is used, add a normally distributed random number with a zero mean and standard deviation of 0.7341 to the expected value before raising it to power two. If Eqns (11)...(14) are used, calculate parameters α and γ of the beta function by using the predicted values on $d_c(n)$, and s_d and select randomly one diameter from the resulting distribution.

(4) Calculate the heights of the trees by Eqn (15).

The method described above is most reliable in the eastern parts of Finland on the typical growing sites of Scots pine. The procedure assumes that the harvests and other treatments affecting the spatial pattern of trees are older that five years.

Fig. 10 shows a few model stands generated by the above procedure. The stocking is 1500 trees/ha in all the cases and the desired basal area 20 m²/ha (the realized basal area may differ from that). It can be seen that in regular tree distributions the two ways to generate diameters produce clearly different results. Method 1, which uses Eqn (10), gives much greater variation in diameter than the use of beta function as a local diameter distribution.

The study material did not contain very regular spatial distributions, and the models may thus be unreliable in such stands
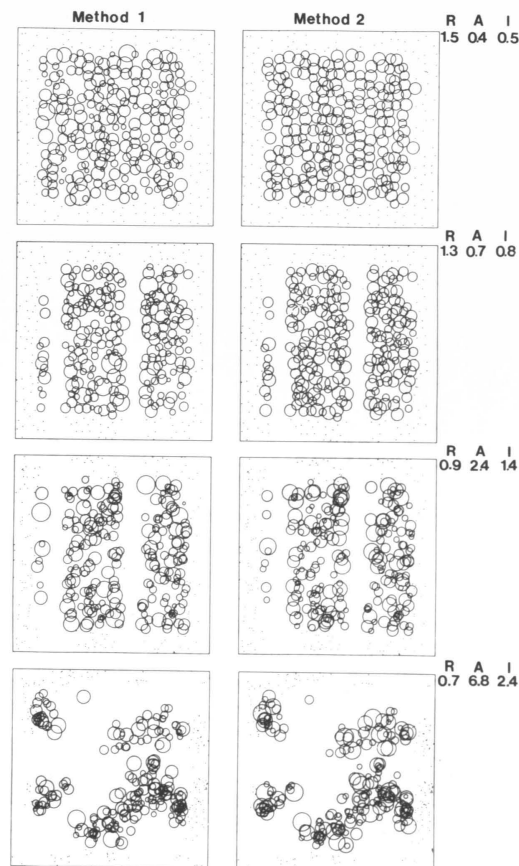

Fig. 10. Examples of stands generated by Eqn (10) (Method 1) and Eqns (11)...(14) (Method 2). In all cases N_s (mean stocking) is 1500 trees/ha and G_s (basal area) is 20 m²/ha.

with both methods. The beta distribution may underestimate the size variation in a regular stand and Method 1 overestimate it. As Väliaho (1971) pointed out, regular spatial distribution does not necessarily lead to a uniform tree size. Size differences appear before the onset of the competition between trees and are thus partly independent of it. On the other hand, a grouped spatial distribution does not necessarily create large variation in the tree size, because the competition can be constant also in aggregated spatial patterns.

5. Discussion

The study presents two ways of simulating tree dimensions so that they are related in a realistic manner to the spatial pattern of tree locations. Both methods predict the local diameter distribution in the spot of each tree and then select randomly one diameter from that distribution.

The first method assumes that the local distribution of the square root of diameter is normal with a constant variance. This means that the diameter distribution of a subarea is between log-normal and normal. These assumptions are not valid in all situations: the local diameter distribution can also be bimodal or clearly log-normal as well (see Figs. 3 and 7). At the stand level the result can be bimodal or of any other type also with Equation (10), depending on the extent of the subareas with different densities. The assumption of constant variance with a given prediction of Equation (10) is not, however, valid, because the variance is also affected e.g. by the spatial distribution of trees. Because normal distribution does not have lower and upper limits, Method 1 produces some abnormally low and high diameters. This shortcoming could be avoided by cutting away part of the distribution.

Owing to these shortcomings of the method based on Equation (10) and a stochastic term, the use of the beta function as a local diameter distribution seems to be a more realistic alternative to generate diameters. This method usually underestimates the range of variation in diameter, if stochastic variation is not added to the predicted minimum and maximum diameter. This does not always happen, but depends on the scale in which the variation in stand density occurs. If the areas of low and high density are large and distinct, also the

estimated lower and upper limits of the diameter distribution vary considerably.

The most obvious use of the models presented in this study is to assist in the generation of artificial stands for simulation studies. One thing that restricts the use of the generated stands is that treatments were not taken explicitly into consideration when constructing the models, although the relation between the spatial pattern, tree size and stem form is affected especially by thinnings. The omission of treatments also decreases the degree of determination of the models, because the trees in different plots of the study material have had a different time period to respond to the increase in the growing space by improving growth.

The fact that cuttings partly destroy the linkage between spatial distribution and that of tree dimensions could have been avoided by using only unthinned stands in the analysis. Stands generated by these models could then be thinned (with harvest roads if desired) and the dimensions of the remaining trees increased by spatial growth models. This way of widening the variety of the generated model stands is of course applicable with the presented models as well. It should be noted, however, that at least the thinnings from below change the spatial distribution in a predictable manner, since they leave the more or less regularly situating big trees (Terborgh 1984) and remove mainly small individuals whose spatial distribution is more irregular. Thinnings affect the spatial distribution of trees in the same way as competition: they shift the spatial pattern towards more regular distributions (Leps and Kindlmann 1987) and decrease the variation in diameter.

References

Adlard, P.G. 1974. Development of an empirical competition model for individual trees within a stand. In: Fries, G. (ed.). Growth models for tree and stand simulation. Rapp. Uppsats. Instn. Skogsprod. Skogshögsk. 30: 22–37.

Bucht, S. 1981. Effekten av några olika gallringsmönster på beståndsutveklingen i tallskog. Summary: The influence of some different thinning patterns on the development of Scots pine stands. Rapp. Instn. Skogsskötsel. Lantbruksniv. 4. 275 p.

Clark, P.J. & Evans, F.C. 1954. Distance to nearest neighbor as a measure of spatial relationships in populations. Ecology 42: 445–453.

Ek, A. & Monserud, R. 1974. Trials with program FOREST: Growth and reproduction simulation for mixed species even- or uneven-aged stands. In: Fries, G. (ed.). Growth models for tree and stand simulation. Rapp. Uppsats. Instn. Skogsprod. Skogshögsk. 30: 56–73.

Eriksson, L. 1976. Konkurrensmodeller för enskilda träds tillväxt efter röjning. Rapp. Uppsats. Instn. Skogstek. Skogshögsk. 99. 85 p.

— 1977. Simulering av beståndsutveklling efter röjning. Summary: Simulation of stand development after cleaning. Rapp. Uppsats. Instn. Skogstek. Skogshögsk. 108. 67 p.

Fisher, R.A, Thornton, H.G. & Mackenzie, W.A. 1922. The accuracy of the planting method of estimating the density of bacterial populations, with particular reference to the use of Thornton's agar medium with soil samples. Ann. Appl. Bot. 9: 325–359.

Hopkins, B. 1954. A new method for determining the type of distribution of plant individuals. Ann. Biology N.S. 18: 213–227.

Kellomäki, S., Hänninen, H., Kolström, T., Kotisaari, A. & Pukkala, T. 1987. A tentative model for describing the effects of some regenerative processes on the properties on natural seedling stands. Tiivistelmä: Alustava malli siementymisen, taimien synnyn ja kasvun vaikutuksesta taimikon rakenteeseen. Silva Fenn. 21(1): 1–10.

Klier, G. 1969. Matematisch-statistische Untersuchungen zur Verteilung der Bäume im Bestand. Wissenschaftliche Zeitschrift der Technischen Universität Dresden. Vol. 18(5): 1061–1065.

Kilkki, P. & Päivinen, R. 1986. Weibull function in the estimation of the basal area dbh-distribution. Seloste: Weibull-funktio pohjapinta-alan läpimittajakauman estimoinnissa. Silva Fenn. 20(2): 149–156.

Leps, J. & Kindlmann, P. 1987. Models of the development of spatial pattern of an even-aged plant population over time. Ecological Modelling 39: 45–57.

Lundell, S. 1973. En modell för simulering av volymtillväxt i teoretiska bestånd. Rapp. Uppsats. Instn. Skogskötsel. 2. 64 p.

Matérn, B. 1960. Spatial variation. Medd. Statens SkogsfoskInst. 49(5): 1–144.

Mielikäinen, K. 1978. Puun kasvun ennustettavuus. Abstract: Predictability of tree growth. Folia For. 363. 15 p.

— 1980. Mänty-koivumetsiköiden rakenne ja kehitys. Summary: Structure and development of mixed pine and birch stands. Commun. Inst. For. Fenn. 99(3). 82 p.

Nyyssönen, A. & Mielikäinen, K. 1978. Metsikön kasvun arviointi. Summary: Estimation of stand increment. Acta For. Fenn. 163. 40 p.

Perry, D.A. 1985. The competition process in forest stands. In: Cannell, M.G.R. & Jackson, J.E. (eds.). Trees as crop plants. Institute of Terrestrial Ecology. Natural Environment Research Council. p. 481–506.

Pukkala, T. 1988. Effect of spatial distribution of trees on the volume increment of a young Scots pine stand. Tiivistelmä: Puiden tilajärjestyksen vaikutus nuoren männikön tilavuuskasvuun. Silva Fenn. 22(1): 1–17.

— 1989. Methods to describe the competition process in a tree stand. Scand. J. For. Res. 4: 187–202.

— & Kolström, T. 1987. Competition indices and the prediction of radial growth in Scots pine. Tiivistelmä: Kilpailutekijät ja männyn sädekasvun ennustaminen. Silva Fenn. 21(1): 55–67.

Päivinen, R. 1980. Rinnankorkeusläpimittajakauman estimointi ja siihen perustuvia puustotunnusten estimointimenetelmiä. University of Helsinki, Department of Forest Inventory. Mimeograph. 92 p.

— 1987. Metsän inventoinnin suunnittelumalli. Summary: A planning model for forest inventory. University of Joensuu. Publ. Sci. 11. 179 p.

Saramäki, J. 1977. Ojitettujen turvemaiden hieskoivikoiden kehitys Kainuussa ja Pohjanmaalla. Summary: Development of white birch (Betula pubescens Ehrh.) stands on drained peatlands in northern Central Finland. Commun. Inst. For. Fenn. 91(2). 52 p.

Terborgh, J. 1985. The vertical component of plant species diversity in temperate and tropical forests. Am. Nat. 126: 760–776.

Tomppo, E. 1986. Models and methods for analyzing spatial pattern of trees. Seloste: Malleja ja menetelmiä puiden tilajärjestyksen analysoimiseksi. Commun. Inst. For. Fenn. 138. 65 p.

Väliaho, H. 1971. The effect of competition on the structure of seedling stands. Seloste: Kilpailun vaikutus taimistojen rakenteeseen. Commun. Inst. For. Fenn. 74(1). 27 p.

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Appendix 1. Stand characteristics of the sample plots. Trees with dbh at least 3 cm included. V = stand volume. Mean diameter (d_g) and mean height (h_g) are weighted by tree basal area.

No.	trees /ha	G m²/ha	V m³/ha	d_g cm	h_g m	R	A	I	No.	trees /ha	G m²/ha	V m³/ha	d_g cm	h_g m	R	A	I
1	1097	23	172	20	16	1.11	1.09	1.01	26	553	16	143	22	19	1.14	0.63	1.03
2	756	20	148	21	15	1.07	1.56	1.30	27	871	29	287	31	22	1.08	1.05	1.03
3	1676	22	151	16	14	0.85	4.88	1.81	28	622	16	141	27	18	1.01	1.10	1.19
4	2019	23	157	15	13	0.90	3.64	1.36	29	1182	13	71	14	11	1.06	1.65	1.13
5	1029	22	197	20	20	1.14	1.11	1.24	30	960	22	194	19	18	1.31	0.63	0.76
6	686	21	185	21	19	1.13	1.03	0.98	31	840	22	178	20	16	1.19	0.76	0.97
7	483	21	194	25	20	1.13	0.51	0.79	32	994	16	117	17	15	1.05	0.96	1.12
8	375	25	268	37	24	1.13	0.70	0.90	33	356	26	270	31	23	1.22	0.82	0.69
9	528	32	315	35	22	1.06	1.03	1.24	34	1257	15	104	14	14	1.22	0.56	0.75
10	792	27	259	23	20	1.30	0.61	1.03	35	906	20	162	18	17	1.29	0.53	0.85
11	584	20	185	22	19	1.27	0.77	0.78	36	584	14	105	20	16	1.23	0.58	0.85
12	509	27	284	32	23	1.14	0.87	1.30	37	1480	9	48	11	10	0.77	5.28	1.49
13	578	29	311	28	23	1.25	0.43	0.88	38	1829	21	112	15	10	1.10	2.11	1.09
14	1509	21	145	15	14	1.27	0.42	0.99	39	3627	13	49	8	6	1.07	2.92	1.20
15	978	20	175	26	18	1.09	1.37	1.19	40	2533	20	140	14	14	0.90	1.86	1.06
16	2126	22	143	14	13	0.95	1.37	1.28	41	1051	19	154	17	16	1.29	0.61	1.16
17	844	18	109	18	12	1.16	0.79	0.63	42	2183	23	169	16	15	1.04	1.08	1.60
18	408	25	241	31	21	1.06	1.11	0.78	43	612	13	101	20	16	0.94	3.13	1.26
19	1520	16	104	14	13	1.17	1.28	0.78	44	1269	20	164	20	17	1.02	0.83	1.07
20	2000	16	96	12	11	1.18	1.11	0.91	45	702	20	180	22	19	1.23	0.58	0.91
21	516	27	271	28	22	1.33	0.40	0.74	46	727	21	190	21	19	1.21	0.65	0.83
22	190	11	128	30	24	1.02	1.19	0.68	47	571	20	194	24	20	1.20	0.45	0.86
23	1863	12	47	10	7	1.27	0.97	0.87	48	2957	19	84	10	8	1.32	0.52	0.91
24	471	15	127	22	18	1.21	1.04	1.15	49	1157	18	132	18	15	0.94	0.90	1.01
25	853	14	95	17	14	1.16	0.76	1.04	50	3827	23	116	10	9	1.12	0.66	0.91