Predicting the Growth Response to Thinning for Scots Pine Stands Using Individual-Tree Growth Models

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Individual-tree growth models for diameter and height, and a model for the cylindrical stem form factor are presented. The aims of the study were to examine modelling methods in predicting growth response to thinning, and to develop individual-tree, distance-independent growth models for predicting the development of thinned and unthinned stands of Scots pine (*Pinus sylvestris* L.). The models were constructed to be applicable in simulation systems used in practical forest management planning. The models were based on data obtained from eleven permanent thinning experiments located in even-aged Scots pine stands in southern and central Finland.

Two alternative models were developed to predict tree diameter growth in thinned and unthinned stands. In the first model, the effect of stand density was described using stand basal area. In the alternative model, an explicit variable was incorporated referring to the relative growth response due to thinning. The magnitude of the growth response was expressed as a function of thinning intensity. The Weibull function was employed to describe the temporal distribution of the thinning response. Both models resulted in unbiased predictions in unthinned and in moderately thinned stands. An explicit thinning variable was needed for unbiased growth prediction in heavily thinned stands, and in order to correctly predict the dynamics of the growth response.

In the height growth model, no explicit thinning variable referring thinning was necessary for growth prediction in thinned stands. The stem form factor was predicted using the model that included tree diameter and tree height as regressor variables. According to the results obtained, the information on the changes in the diameter/height ratio following thinning is sufficient to predict the change in stem form.

Keywords growth modelling, individual-tree, distance-independent, thinning, stem form, *Pinus sylvestris*.
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List of Symbols and Definitions

Stand variables

H_{dom} = Stand dominant height defined as: average height of the 100 thickest trees per hectare, m
IH_{dom} = Five-year increment of dominant height, m
H_{100} = Site index, m (base age 100 years, calculated using models developed by Vuokila and Väliaho 1980)
H_g = Mean height, weighted with basal area, m
D_g = Mean diameter, weighted with basal area, cm
D_{dom} = Stand dominant diameter defined as: average diameter of 100 thickest trees per hectare, cm
G = Stand basal area over bark, $m^2\ ha^{-1}$
I = Thinning intensity, defined as: (G, pre-thinning – G, post-thinning) / G, post-thinning
T = Time elapsed from thinning, years
D_{502}... D_{558} = Categorical variables referring to experimental stands

Tree variables

d = Diameter at breast height, over bark, cm
$d_{6.0}$ = Diameter at 6 m height, over bark, cm
i_{d5} = Five-year increment in tree diameter, cm
g = Tree basal area at breast height, m^2
h = Height, m
i_{h5} = Five-year increment in tree height, m
v = Tree volume, m^3
GL = Basal area of trees (over bark) larger than subject tree, $m^2\ ha^{-1}$
cr = Tree crown ratio, defined as: length of live crown/total tree height
$f_{1.3} = v / gh$ = Cylindrical stem form factor

Other definitions

$a_0, a_1 ... a_7, b, c$ = Parameters
e = Error term
y_i = Observed value of i:th observation
$\hat{y}_i$ = Predicted value of i:th observation
n = Number of observations

RMSE $= \left[\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 / n\right]^{0.5}$ (Root mean square error)

RMSE$_r$ $= \left[\sum_{i=1}^{n}\left[(y_i - \hat{y}_i)/\hat{y}_i\right]^2 / n\right]^{0.5}$ (Relative root mean square error)

Absolute bias $= \sum_{i=1}^{n}(y_i - \hat{y}_i)/n$

Relative bias $= \sum_{i=1}^{n}\left[(y_i - \hat{y}_i)/\hat{y}_i\right]/n$

1 Introduction

In Finnish forestry, thinning from below is the most widespread treatment applied in silviculture. The regulation of stand density with the help of intermediate thinnings has been based on both silvicultural and economical aspects. During the recent years, thinning costs have increased. Especially the first commercial thinning of young stands has become less profitable, and this has resulted in changes in thinning schedules. Therefore, it has become increasingly important to be able to forecast the impacts of alternative thinning schedules on the future development of forests. The simulation systems applied in forest management planning should be capable of reliably predicting stand development regardless of the thinning treatment applied.

Tree diameter growth is known to be affected by stand density. Thinning decreases stand density abruptly, and this has a strong impact on tree growth. Growth response following thinning is the result of (i) increased growing space, (ii) the fertilization effect provided by the non-harvested parts of felled trees, and (iii) the selection effect (Hägglund 1981). The latter effect means that trees retained in stands subjected to thinning from below have grown better before thinning compared to those removed in thinning.

In many growth simulators used in forest management planning, tree growth is predicted using models that do not include any explicit thinning effect (e.g. Belcher et al. 1982, Wykoff et al. 1982, Burkhart et al. 1987, Ojansuu et al. 1991). Such models are based on the assumption that the thinning response can be described through stand characteristics, which are affected by stand density and will change due to thinning.

An alternative method for predicting the effect of thinning is to incorporate an explicit thinning variable in the growth model. This approach has been justified by a hypothesis, according to which an abrupt change in stand density, caused by thinning, changes the effect of stand density on tree growth. Consequently, the effect of stand density in two stands of equal stand density is different in the stand that has been recently thinned compared to the stand where the trees have initially been more widely spaced. There is a group of models in which the thinning response is expressed explicitly in terms of categorical variables (Harrison et al. 1986, Söderberg 1986, Shafii et al. 1990). These models are capable of predicting the magnitude of the total growth response to thinning over the predicted growth period. This approach is not, however, flexible enough to give any information about the temporal distribution of the response. Jonsson (1974) has developed a model for the relative thinning response in tree diameter growth that is capable of predicting both the magnitude and the temporal distribution of the response. Jonsson used information about diameter increment in unthinned and thinned stands during the growth period prior and subsequent to thinning in modelling the thinning response. In addition to growth models, the magnitude and duration of the thinning response has been incorporated in models predicting the crown ratio of trees (Short and Burkhart 1992, Hynynen 1995).

Increment in dominant height is known to be fairly insensitive to stand density and intermediate thinnings as long as stands are thinned from below (e.g. Assmann 1970, Hägglund 1974, Clutter et al. 1983, Vuokila and Väliaho 1980). In individual-tree growth models, height growth is generally expressed as a function of the increment in the stand dominant height and the relative, or absolute, tree size, without any explicit thinning response variable (e.g. Wykoff et al. 1982, Arney 1985, Burkhart et al. 1987, Ojansuu et al. 1991).

Stem form is strongly affected by stand density. Differences in the thinning response in tree diameter and height growth result in changes in the stem form; this is well documented in many growth and yield studies (Vuokila 1960, Assmann 1970, Söderberg 1986, Valinger 1990). Most growth simulators based on individual-tree models predict tree growth either by means of tree diameter/basal area growth models (Belcher et al. 1982) or diameter/basal area and height growth models (Wykoff et al. 1982, Arney 1985, Burkhart et al. 1987, Ojansuu et al. 1991). In all these simulators, tree volume is predicted with static volume equations. This kind of simulation procedure implies two assumptions concerning the prediction of stem form and stem volume. First, volume equations are assumed to be applicable in volume prediction for all trees regardless of the thinning treatment applied. Second, the change in stem form due to thinning, and in more general due to change in stand density, can be explained by the change in the d/h ratio.

The aims of this study were to examine the modelling methods used in predicting the growth response to thinning, and to develop individual-tree, distance-independent growth models for predicting the development of thinned and unthinned stands of Scots pine. The models were constructed to be applicable in the simulation systems used in practical forest management planning. The input of the models were determined to be consistent and compatible with the information available in practical forest inventory data.

In modelling the growth response to thinning, there were three specific areas of interest. First, in modelling diameter growth, the goal was to determine whether an explicit thinning variable needs to be incorporated in the model for unbiased growth prediction in thinned stands. Second, the effects of thinning on the development of dominant height increment as well as on the growth of individual tree were examined. Third, a further aim was to analyse whether the effect of thinning on tree stem form development can be explained by changes in the diameter/height ratio.

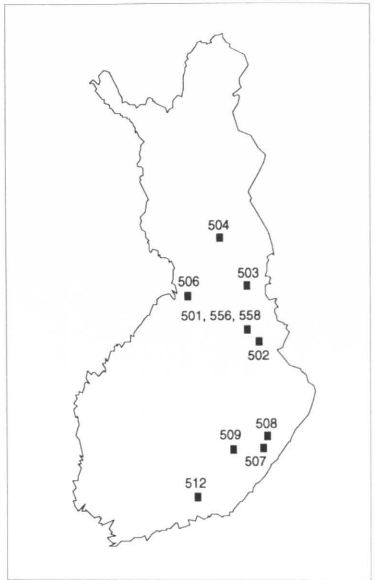

Fig. 1. Location of the experimental stands.

2 Study Material

2.1 Modelling Data

Data were obtained from permanent sample plots established in experimental stands of Scots pine (*Pinus sylvestris* L.). The study material consisted of eleven even-aged stands growing on mineral soils and located in southern and central Finland (Fig. 1). The experiments were established by the Finnish Forest Research Institute in the early 1970s with the purpose of studying the effects of varying thinning intensities and nitrogen fertilization on the growth and yield of stands of Scots pine.

Prior to the establishment of the experimental plots, each stand had been thinned to an average density of 2355 trees/ha at the seedling stage. At the time the experiments were established, all the stands had reached the stage of the first commercial thinning. Stand age in the study material varied between 29 and 56 years, and the mean stand height varied between 10.0 m and 15.2 m (Table 1).

The effects of three levels of thinning intensities and three levels of nitrogen fertilization were studied using a factorial experimental design. In the experimental stands, each treatment was applied on one rectangular sample plot 1000 m^2 in size except for one stand, in which two control plots were established.

Only the unfertilized sample plots were included in the analyses conducted in the course of this study. Once the initial measurements had been carried out, one third of the sample plots was left unthinned, one third was thinned moderately (30 % of the stem number were removed) and one third was thinned heavily (60 % of the stem number were removed). In moderately thinned plots, the second thinning was carried out ten years after the first thinning by again removing 30 % of the initial stem number. On these sample plots, only the data from the measurement instances prior to the second thinning were included in the analyses.

Table 1. Mensurational characteristics of the study material.

Exp. no	Age, years	Site type [1)]	H_{100}, m	H_g, m	D_g, cm	G, m^2 ha^{-1}	Stem number, no ha^{-1}	No. of sample plots	No. of sample trees / sample plot Mean (Min.– Max.)
501	40	EVT	23.4	10.0	10.4	18.2	3003	3	42.7 (40–46)
502	55	EVT	20.5	10.9	12.4	17.9	2084	3	48.7 (47–51)
503	41	VMT	24.2	11.8	14.3	23.9	1858	3	45.7 (44–48)
504	45	VMT	21.5	11.9	14.1	21.7	1800	3	39.3 (29–49)
506	56	VT	22.1	14.7	17.6	21.7	1118	4	43.2 (32–49)
507	38	VT	25.6	10.7	11.3	22.8	3070	3	39.0 (21–51)
508	39	VT	24.9	10.7	11.0	22.9	3148	3	39.0 (22–52)
509	29	MT	28.8	10.7	13.3	25.6	3081	3	42.3 (32–50)
512	48	VT	21.0	10.6	11.8	20.1	2423	3	35.0 (29–42)
556	40	EVT	23.8	10.3	10.8	18.7	2762	3	47.3 (46–49)
558	44	EVT	26.5	15.2	15.9	25.9	1560	3	37.3 (29–45)

[1)] According to Cajander (1909)

The stands were measured at five-year intervals over a study period of 15 years. All the trees on the sample plots were measured for their breast height diameter. In the first measurement instance, an average of 42 sample trees were selected from each sample plot and used throughout the study period. During the stage of selecting the sample trees, the probability of a tree to be selected was proportional to its diameter and independent of its location on the sample plot. Two thirds of the sample trees were thicker than the stand's average diameter. The height and crown height were measured for every sample tree. Crown height was defined as the height above ground of the lowest live contiguous branch whorl. In addition to breast height diameter, also diameter at six metres and diameters at the relative heights of 2.5 %, 10 %, 30 % and 50 % along the stem were measured.

The stand-level characteristics of the growing stock were calculated using a software package for computing stand and tree characteristics (KPL), developed at the Finnish Forest Research Institute (Heinonen 1994). Height information obtained for sample trees was generalised with help of Näslund's (1937) height curve to apply to the tallied trees. Sample tree volumes were calculated using the simultaneous equations developed by Laasasenaho (1982). By using these functions, all the available tree diameter observations at absolute and relative heights along the stem could be used in the stem volume calculations. Volumes for the tallied trees were computed from the sample tree volumes by using smoothing functions. Increments for the tree and stand variables were calculated as differences between the values of the variables at the end and the beginning of the five-year growth periods.

Only sample trees were used in the model development. The study material included 4634 measurements of tree characteristics involving 1579 sample trees located on 34 sample plots. The number of tree diameter growth observations (five-year growth periods) was 3479, and the number of tree height growth observations was 3406.

The effect of annual climatic variation on tree growth was taken into account with the help of annual growth indices provided by Mielikäinen and Timonen (1995). For every five-year growth period, an average index was calculated from the annual growth indices, with which the observed diameter and height growth was divided.

2.2 Test Data

Data from permanent thinning experiments established by the Finnish Forest Research Institute (Vuokila 1987) were used as the independ-

Table 2. Mensurational characteristics of the test material.

Exp. no	Age, years	Site type [1]	H_{100}, m	H_g, m	D_g, cm	G, $m^2\,ha^{-1}$	Stem number no ha^{-1}
6	77	VT	20.3	17.9	20.3	23.0	795
42	35	VT	27.5	13.0	16.1	19.6	1075
63	24	MT	26.6	7.9	12.2	16.7	1703
65	36	VT	30.7	14.5	16.0	30.0	1919
541	52	VT	26.0	17.7	21.9	18.6	527
542	62	VT	24.0	17.2	20.3	16.5	551

[1] According to Cajander (1909)

ent test material in model validation. The test data included 3551 trees with 5-year growth observations covering 24 sample plots located in six stands in southern Finland (Table 2). Nine of the sample plots were unthinned. The thinning intensity among the thinned sample plots varied between 15–55 % (of the stand basal area removed). The average thinning intensity was 27 %. The study period covered 5–13 years after thinning.

The sample plots providing the test data were measured, and the sample trees on these plots were selected in the same manner as with the modelling data. Also, the calculation of tree and stand variables was done in a similar manner, except for the calculation of sample tree volumes. These were calculated using the volume equations provided by Laasasenaho (1982), based on tree diameters at breast height and at 6 m height, and on tree height, because the sample trees were not measured for their diameters at relative heights.

3 Diameter Growth Model

3.1 Modelling Approach

Tree diameter growth was assumed to be affected by site fertility, the amount of the growing biomass, and the net-assimilation efficiency of the biomass (Jonsson 1969, Hägglund et al. 1979). It was further assumed that the effects of the different growth factors interact multiplicatively (Baule 1917).

Site fertility was expressed using the site index (H_{100}) calculated with the equations provided by Vuokila and Väliaho (1980). The amount of the growing biomass was described in terms of tree diameter and the crown ratio. The net assimilation efficiency of the growing biomass was assumed to be affected by stand density, described in terms of the stand basal area, relative positions of trees in the stand, described in terms of the basal area of trees larger than the subject tree, and the phase of stand development, described in terms of the stand dominant height.

Two separate diameter growth models were developed. In the first model, the effect of thinning on tree growth was assumed to be taken into account by including the basal area of the growing stock as a regressor variable referring to the actual stand density. In the second model, the effect of thinning on tree growth was incorporated explicitly in the model by using a variable accounting for the thinning intensity and time interval since thinning. In both models, five-year diameter growth, over bark, was used as a dependent variable.

Because of the hierarchical data structure, there was temporal autocorrelation between successive observations made of a single tree, and there was spatial autocorrelation between observations made of trees on the same sample plot. The effect of autocorrelation was not taken into account in the parameter estimation of the models, because autocorrelation does not generally affect the unbiasedness of models. Ordinary Least Squares (OLS) estimation was applied in the parameter estimation of all the models. Parameters of the nonlinear regression models were estimated using the NLIN program of the SAS software package (SAS Institute Inc., 1989) and applying Marquardt's method with the convergence criterion set to 10^{-8}.

3.2 Model without Explicit Thinning Response Variable

In the first diameter growth model, it was assumed that the thinning effect would be reflected in tree growth through the actual stand basal area and through the variables affected by stand density. Therefore, no explicit thinning variable was included in the model. The analysis of the data resulted in the following model

$$i_{d5} = a_0 d^{a_1} cr^{a_2} \exp\left(a_3 d^2 + a_4 GL^2\right) H_{dom}^{a_5} H_{100}^{a_6} G^{a_7} + e \quad [1]$$

where

i_{d5} = Five-year increment of tree diameter, cm
d = Tree diameter at breast height, over bark, cm
cr = Tree crown ratio, defined as: length of live crown/total tree height
GL = Basal area, over bark, of trees larger than the subject tree, $m^2\,ha^{-1}$
H_{dom} = Stand dominant height defined as: average height of 100 thickest trees per hectare, m
H_{100} = Site index, m (base age 100 years, calculated using models provided by Vuokila and Väliaho 1980)
G = Stand basal area, over bark, $m^2\,ha^{-1}$
$a_0, a_1 \ldots a_7$ = Parameters
e = Error term

The formulation of the effect of the stand basal area (G^{a_7}) was chosen, although it leads to illogical model behaviour when the stand basal area is close to 0 $m^2\,ha^{-1}$. Despite this structural weakness, the applied expression proved to describe the effect of the stand basal area in the modelling data better than the other examined transformations of the stand basal area. Model behaviour is logical within the range of basal area variation of the modelling data (G > 9.5 $m^2\,ha^{-1}$).

Table 3. Parameter estimates of diameter growth model [1].

Parameter	Estimate	Asymptotic Std. Dev.
a_0	0.0504	0.0137
a_1	0.7917	0.0733
a_2	0.5557	0.0439
a_3	–0.0010	0.0001
a_4	–0.00075	0.00008
a_5	–0.6470	0.0654
a_6	1.4995	0.0606
a_7	–0.4349	0.0256
$\bar{i}_{d5}$	1.1479	
RMSE	0.4046	
Observations	3479	

The parameter estimates of model [1] were obtained with the OLS estimation (Table 3). The autocorrelation between the observations of the modelling data does not affect the parameter estimates. However, the standard error of the estimates obtained with OLS are likely to be too small.

Residual analysis of model [1] showed a slight increase in the error variance with increasing predicted growth (Fig. 2a). However, there were no trends in the residuals with respect to the predicted diameter growth (Fig. 2a) or with respect to the regressor variables of the model. Residuals plotted against thinning intensity showed that model behaviour in general was satisfactory in unthinned and moderately thinned stands (Fig. 2b). However, in heavily thinned stands, with more than 50 % of stand basal area removed, significant biases were observed. In these stands, the mean growth was underpredicted on average by 17.1 % over the entire 15-year study period.

During the first five-year growth period following thinning, model [1] overpredicted growth, except in the case of the heavily thinned stands, in which the model resulted in a small underprediction (Fig. 2b). During the second and third growth periods, the model resulted in a noticeable underprediction in heavily thinned stands, but also in a slight underprediction in unthinned and moderately thinned stands. In heavily thinned

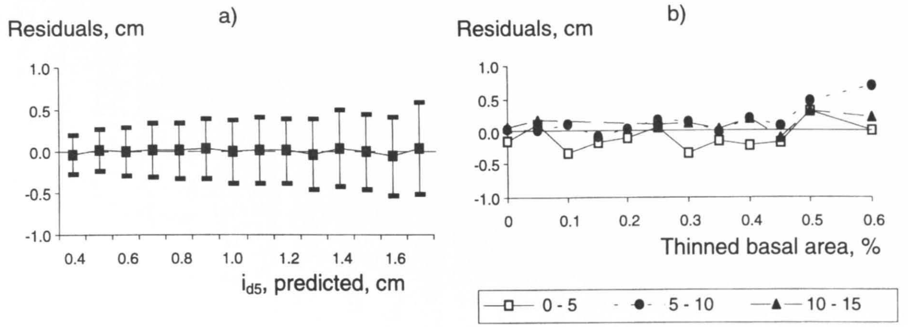

Fig. 2. Mean residuals (± standard deviation of the residuals) of the diameter growth model [1] with respect to predicted diameter growth (a), and mean residuals during the five-year growth periods with respect to thinning intensity (b).

stands, the bias was at its highest during the second growth period, 5–10 years after thinning.

The predictive capability of the tree crown ratio was examined by fitting a model similar to [1], but from which the crown ratio (*cr*) had been excluded. The root mean square error (RMSE) of the model after excluding *cr* was 0.4140, i.e. 2.3 % greater than the RMSE of model [1], which was 0.4046. Removing the crown ratio from model [1] did not change the model behaviour in regard to thinning intensity.

3.3 Model with Thinning Response Variable

An alternative diameter growth model was developed with the effect of thinning explicitly incorporated in it. Tree diameter growth in a thinned stand can be expressed as a product of a reference growth and thinning response function

$$i_{d5} = F_1(ref) \cdot F_2(thin) \qquad [2]$$

Reference growth ($F_1(ref)$) accounts for the factors affecting tree growth in unthinned stands. The thinning response function ($F_2(thin)$) predicts the relative growth response following thinning, and the reference growth is multiplied with this. The model structure is similar to that of the growth model for fertilized Scots pine stands developed by Hynynen (1993).

The Weibull function was applied in modelling the temporal distribution of the thinning response. It was assumed that tree growth responds to thinning without any delay. Thus, a two-parameter Weibull function was applied in the model. The integral of the Weibull function equals one. In order to get varying magnitudes of response as the results of different thinning intensities, the Weibull function was scaled by multiplying it with a variable expressed as a function of the thinning intensity.

Table 4. Parameter estimates of diameter growth model [3].

Parameter	Estimate	Asymptotic Std. Dev.
a_0	0.1769	0.0454
a_1	0.5693	0.0659
a_2	0.4737	0.04242
a_3	–0.00070	0.00014
a_4	–0.00094	0.00008
a_5	–0.9694	0.0661
a_6	1.0796	0.0539
a_7	7.7395	0.5512
b	13.4054	0.5314
c	2.4828	0.1385
$\hat{\bar{i}}_{d5}$	1.1473	
RMSE	0.3842	
Observations	3479	

As the result of analysis, the following model was developed

$$i_{d5} = a_0 d^{a_1} cr^{a_2} \exp\left(a_3 d^2 + a_4 GL^2\right) H_{dom}^{a_5} H_{100}^{a_6} \cdot F_2(thin) + e$$

in which

$$F_2(thin) = 1 + a_7 I \left(\frac{c}{b}\right)\left(\frac{T}{b}\right)^{(c-1)} \exp\left(-\left(\frac{T}{b}\right)^c\right) \qquad [3]$$

where

I	=	Thinning intensity, defined as: (G, pre-thinning – *G*, post-thinning) / *G*, post-thinning
T	=	Time elapsed from thinning, years
$a_0, a_1,..,a_7, b, c$	=	Parameters

In model [3], the regressor variables used in predicting reference growth are the same as in model [1] except for the stand basal area. The effect of the stand basal area is reflected in growth through the size of the tree crown ratio, and in thinned stands also through the thinning response function. According to the model, the relative response to thinning is not affected by site, and neither by any stand or tree variables.

Because model [3] predicts the five-year tree diameter growth, also the temporal distribution of thinning response is predicted by five-year periods. Thus, $F_2(thin)$ refers to the average relative growth response during the 5-year growth period in question. Consequently, variable T in model [3] refers to the last year of the 5-year growth period. According to model [3], thinning increases the relative diameter growth without any delay. The response reaches its maximum within a period of 5–10 years after thinning, and levels off by 30 years after thinning (Fig. 3).

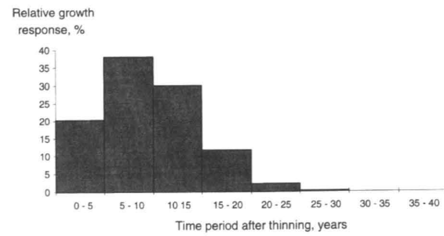

Fig. 3. Temporal distribution of the relative diameter growth response according to thinning response function, $F_2(thin)$, of the model [3].

Including the effect of thinning in the diameter growth model improved the accuracy of the model compared to model [1], in which the thinning effect is implicitly included through the actual stand basal. The root mean square error in model [1] was reduced from 0.4046 (Table 3) to 0.3841 (Table 4), i.e. by 5.1 %.

An improvement of the model was also observed in residual analysis (Fig. 4). No bias was

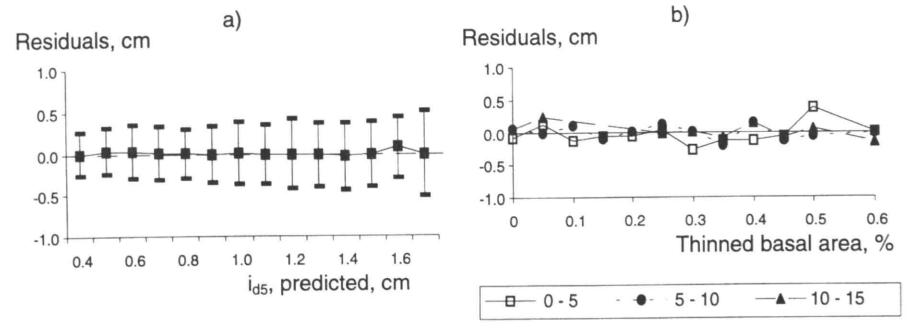

Fig. 4. Mean residuals (± standard deviation of the residuals) of the diameter growth model [3] with respect to predicted diameter growth (a), and mean residuals during the five-year growth periods with respect to thinning intensity (b).

observed with respect to any of the regressor variables. There were no trends in the residuals as regards thinning intensity during any of the successive growth periods (Fig. 4b). Therefore, the model's performance was improved compared to model [1].

The tree crown ratio (*cr*) was a significant regressor variable in model [3] as well as in model [1]. Removing the tree crown ratio from model [3] increased the RMSE from 0.3841 to 0.3910, i.e. by 1.8 %.

4 Height Growth Model

The tree height growth model was developed on the basis of an assumption according to which tree height growth can be represented as the product of potential height growth times a modifier function. This kind of a model structure has been widely applied in growth and yield modelling (e.g. Daniels and Burkhart 1975, Leary 1979, Arney 1985).

Increment in the stand dominant height was regarded as the potential height growth. With reference to earlier studies, it is generally assumed that dominant height increment is not affected by thinning from below (Burkhart et al. 1987, Vuokila and Väliaho 1980). To verify this assumption within the context of the modelling data, a simple regression model for dominant height increment was developed. In the model, dominant height increment was assumed to be affected by site quality, stand age and thinning intensity. The effects of site quality and stand age were taken into account by using categorical variables referring to the experimental stand. The effect of thinning was studied by incorporating a variable referring thinning intensity into the model. The mean annual increment in the stand dominant height over the 15-year study period was employed as the dependent variable of model [4] below.

$$\ln(IH_{dom}) = a_0 + a_1 I + a_2 D_{502} + a_3 D_{503} + \ldots + a_{11} D_{558} + e \qquad [4]$$

where

IH_{dom} = Mean annual increment of stand dominant height over 15-year study period, m

$D_{502} \ldots D_{558}$ = Dummy variables referring to experimental stands

$a_0, a_1 \ldots a_{11}$ = Parameters

Model [4] was fitted to the data including observations of the dominant height increments from every sample plot in the data. The effect of thinning intensity did not prove to be a significant regressor (Table 5). Therefore, it was concluded that thinning intensity does not have any significant effect on the increment of the stand dominant height, which was employed as the height growth potential of an individual tree in a stand.

In developing the height growth model for individual trees, it was supposed that tree growth

Table 5. Parameter estimates of height growth model [4].

Parameter	Estimate	Std. Error	t-value	Prob. > |T|
Intercept	–1.194			
a_1	–0.103	0.129	–0.802	0.431
$\overline{\ln(\hat{I}H_{dom})}$	–1.215			
R^2	0.901			
RMSE	0.129			
Observations	32			

Note: *Intercept* = parameter a_0 + mean of the parameters $a_2, \ldots, a_{11}$.

Table 6. Parameter estimates of tree height growth model [5].

Parameter	Estimate	Asymptotic Std. Dev.
a_1	0.2445	0.0151
a_2	–0.4710	0.0558
a_3	0.8045	0.1804
$\overline{\hat{i}_{h5}}$	1.4794	
RMSE	0.4495	
Observations	3406	

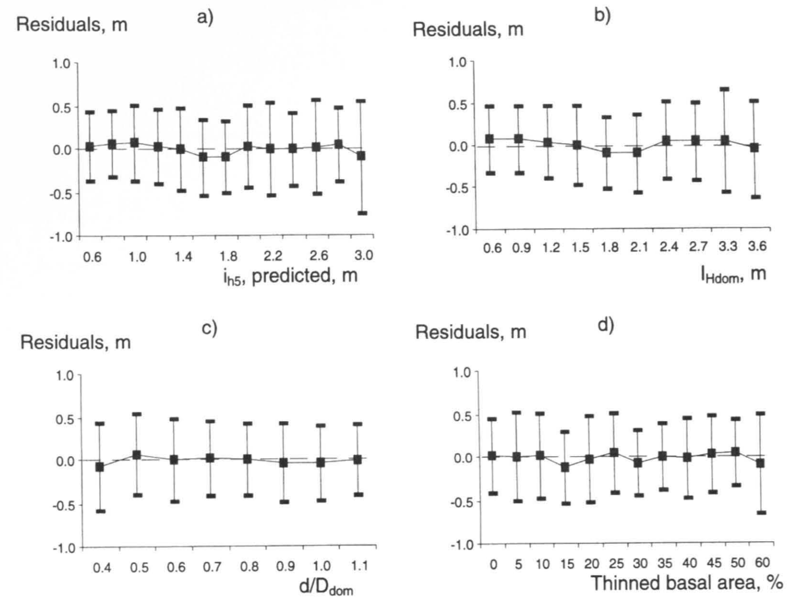

Fig. 5. Mean residuals (± standard deviation of the residuals) of tree height growth model [5] with respect to predicted height growth (a), stand dominant height increment (b), relative tree size (c), and thinning intensity (d).

can be faster or slower than potential growth depending on the relative size of the tree. Relative tree size was described by the ratio between tree diameter at breast height and the stand dominant diameter (d / D_{dom}), the latter being defined as the average diameter of the 100 thickest trees per hectare. Thus, the stand dominant diameter is the arithmetic mean diameter of the trees included in the calculation of the stand dominant height (H_{dom}).

It was further assumed that thinning from below does not directly affect tree height growth, and that stand basal area affects only the growth of suppressed trees. The crown ratio (*cr*) can be considered as an expression of the tree's photosynthetic potential. Therefore, it can be assumed to have an effect on the realization of potential tree growth.

The parameters were estimated using the same method as when estimating the parameters of diameter growth models [1] and [3]. The analysis resulted in the following model for tree height growth (Table 6):

$$i_{h5} = IH_{dom}\left[d / D_{dom}\right]^{\left(a_1 IH_{dom} + a_2 (d/D_{dom})^{a_3}\right)} + e \qquad [5]$$

where

i_{h5} = Five-year increment in tree height, m

IH_{dom} = Five-year increment in dominant height, m

D_{dom} = Stand dominant diameter defined as: average diameter of 100 thickest trees per hectare, cm

a_1, a_2, a_3 = Parameters

The effects of the tree crown ratio and the stand basal area on height growth proved to lack statistical significance as regressors. Thus, they were not included in the final model. Residual analysis showed satisfactory model behaviour in regard to predicted height growth, regressor variables and thinning intensity (Fig. 5). Although the effect of thinning was not incorporated in the model, there were no trends in the residuals with respect to thinning intensity during any of the growth periods.

In fitting the model, the measured increment in the stand dominant height (IH_{dom}) was used as the potential height growth. When applying the model, IH_{dom} can be obtained from the site index equation.

Model [5] is restricted so that trees with diameters equal to the average diameter of the dominant trees will have height growths equal to the increment of the dominant trees. The relationship between tree height growth and relative tree size is of curvilinear form. Starting from the most suppressed trees in a stand, height growth increases with increasing relative tree size, until it reaches its maximum (Fig. 6). After that, growth starts to decrease with increasing relative size. The position of maximum height growth depends on the rate of the dominant height increment.

According to the model, growth of individual trees in stands with rapid dominant height increment (IH_{dom}) is more affected by relative tree size than in stands with slow dominant height increment (Fig. 6). In other words, differentiation in height growth among trees is greatest in stands with rapid height growth. With respect to stand growth dynamics, height growth differentiation is at its highest in young stands.

5 Model for Predicting Stem Form Factor

A measure of tree stem form is needed in addition to the tree diameter and height growth prediction in order to simulate the development of stand and tree volumes. The ratio between tree diameter and total height (d/h) is an indicator of the stem form. Thinning is known to change this ratio, which can be predicted with the help of diameter and height growth models ([3] and [5]).

In this study, the development of a model for predicting the stem form factor was of interest as the aim was to examine whether thinning affects

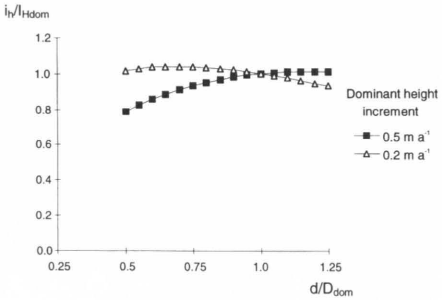

Fig. 6. The relationship between relative height growth of a tree and relative tree size in stands with different rate of dominant height increment.

stem form in a way not explained by the change in the d/h ratio. In order to be able to quantify these kinds of possible effects in the present study material, a model for the stem form was developed.

A static model for the cylindrical form factor was chosen to describe the stem form. The cylindrical form factor is a widely used expression for tree stem form. It is defined as the ratio of the total stem volume to the volume of a cylinder with diameter equal to tree diameter at breast height and height equal to the total height of the tree. The stem volume can be expressed as

$$v = f_{1.3} g h \qquad [6]$$

where
v = Tree volume, m^3
$f_{1.3}$ = Cylindrical form factor
g = Tree basal area at breast height, m^2
h = Tree height, m

The absolute value of the form factor is restricted to between 0 and 1, but excluding small trees with heights close to 1.3 m. In the present modelling data, the smallest measured tree height was 4.5 m. Thus, the model for the stem form factor should inherently result in values between 0 and 1 to facilitate logical model behaviour. The following model structure was employed as the basic structure in model development.

$$f_{1.3} = 1 - \exp(-\Phi(x)), \text{ where } \Phi(x) > 0 \qquad [7]$$

Table 7. Parameter estimates of model for form factor [8].

Parameter	Estimate	Asymptotic Std. Dev.
a_0	0.6390	0.0159
a_1	–0.0263	0.0040
a_2	3.0028	0.2742
a_3	3.7094	0.6233
a_4	–1.2138	0.1010
$\hat{f}_{1.3}$	0.5341	
RMSE	0.0251	
Observations	4634	

Table 8. Parameter estimates of model for form factor [9].

Parameter	Estimate	Asymptotic Std. Dev.
a_0	0.8217	0.0252
a_1	–0.0381	0.0065
a_2	2.4243	0.2733
a_3	6.5014	1.4381
a_4	–1.6114	0.1299
a_5	–0.0638	0.0111
a_6	–0.0021	0.0007
a_7	–0.0020	0.0002
$\hat{f}_{1.3}$	0.5341	
RMSE	0.0248	
Observations	4634	

In model [7], $\Phi(x)$ is a function of measured tree and stand variables. A similar structure has been used earlier in numerous models for the tree crown ratio, which is also a variable restricted to values between 0 and 1 (e.g. Ek and Monserud 1975, Dell et al. 1979, Dyer and Burkhart 1987, and Hynynen 1995).

A simple model for stem form factor was developed based only on the information about tree diameter and height. After fitting the model, its behaviour was studied with respect to other stand and tree variables, including thinning intensity.

As the result of analysis, the following model for the stem form factor was constructed (Table 7):

$$f_{1.3} = 1 - \exp\left[-\left(a_0 + a_1 (d/h)^{a_2} + a_3 h^{a_4}\right)\right] + e \qquad [8]$$

According to model [8], trees with more taper have smaller stem form factors. Furthermore, the form factor decreases with increasing tree size, and this is expressed with total tree height.

The residuals showed no trends with respect to the predictor variables (Fig. 7). Plotted residuals against stand basal area, stand dominant height, as well as against tree crown ratio showed slight trends, but the model resulted in unbiased prediction concerning thinning intensity (Fig. 7f).

In order to improve the model, the effects of the other measured stand and tree variables, in addition to d and h, was examined. Tree crown ratio, stand dominant height, and stand basal area proved to be significant regressors, resulting in the following model:

$$f_{1.3} = 1 - \exp\left[-\left(\begin{array}{l} a_0 + a_1 (d/h)^{a_2} + a_3 h^{a_4} \\ + a_5 cr + a_6 H_{dom} + a_7 BA \end{array}\right)\right] + e \qquad [9]$$

The inclusion of new variables in model [8] removed the biased behaviour with respect to these variables, but improved the precision of the model only slightly by reducing the RMSE by 1.2 % (Table 8).

According to model [9], increases in the tree crown ratio and the stand dominant height impair the stem form; this is consistent with the previous knowledge on these relationships. The slightly negative effect of the basal area ($a_7 < 0$) on the form factor is more difficult to interpret. However, the effect of the stand basal area will also be reflected in the tree crown ratio and the ratio d/h. As the basal area increases, the crown ratio and d/h decrease, and these in turn increase the form factor, and thus improve the stem form. Therefore, the effect of the stand basal area is also implicitly included in the other regressor variables.

On the basis of the stem form factor models [8] and [9], it can be concluded that in the case of both unthinned and thinned stands, the development of the stem form can be predicted without

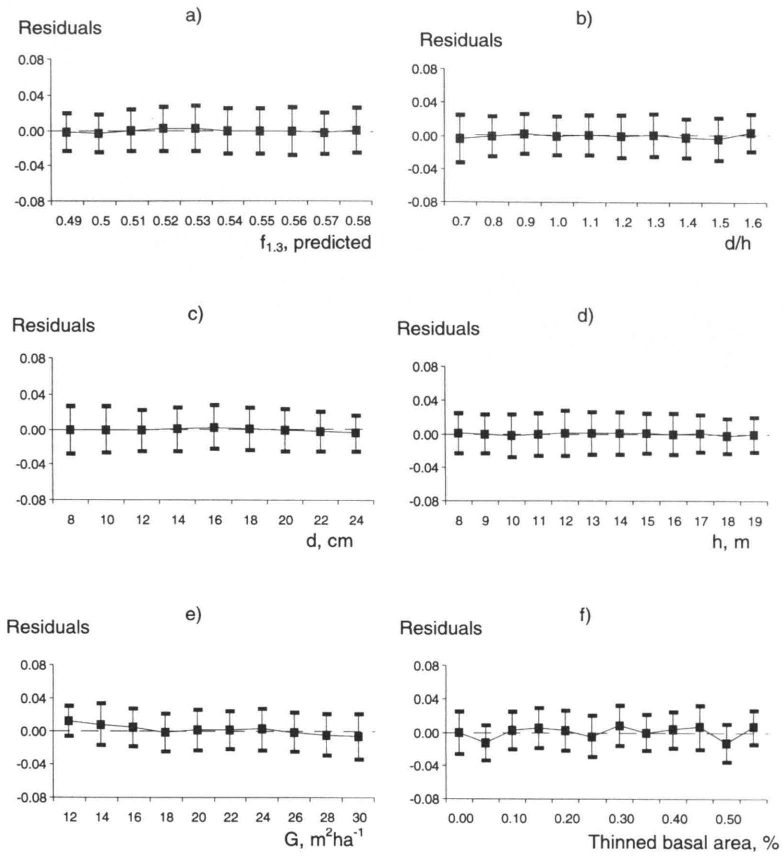

Fig. 7. Mean residuals (± standard deviation of the residuals) of stem form factor model [8] with respect to predicted form factor (a), *d/h* ratio (b), tree diameter (c), tree height (d), stand basal area (e), and thinning intensity (f).

bias by using those tree and stand variables that are used as regressors in models [8] and [9]. No additional information concerning thinning itself is needed. The results of this analysis suggest that the effect of thinning on tree stem form can be explained with adequate accuracy by the change in the *d/h* ratio.

6 Model Validation

6.1 Reliability of Models in Predicting Tree Diameter Growth, Height Growth, and Stem Volume

All the models were tested against independent data as described in section 2.2. Validation was first carried out separately for each model to test their predictive capability. Then, the models were applied together in predicting the stand basal

Table 9. Behavior of the models against the independent test data.

	Diameter growth [1]		Height growth [1]	Tree volume (form factor)
	Model [1]	Model [3]	Model [5]	Model [8]
Observed, mean	1.302 cm	1.302 cm	1.949 m	150.09 dm³
Predicted, mean	1.274 cm	1.221 cm	1.939 m	158.38 dm³
Absolute bias	0.028 cm	0.081 cm	0.011 m	–8.296 dm³
Relative bias	0.0387	0.0878	0.0426	–0.0577
RMSE	0.639	0.647	0.383	13.039
$RMSE_r$	0.716	0.761	0.307	0.067
Observations	3551	3551	3523	8583

[1] Five-year growth period

area and volume increment. The following characteristics were calculated to describe the reliability of the model prediction:

$$\text{Absolute bias} = \sum_{i=1}^{n}(y_i - \hat{y}_i)/n$$

$$\text{Relative bias} = \sum_{i=1}^{n}\left[(y_i - \hat{y}_i)/\hat{y}_i\right]/n$$

$$\text{RMSE} = \left[\sum_{i=1}^{n}(y_i - \hat{y}_i)^2/n\right]^{0.5}$$

= root mean square error

$$\text{RMSE}_r = \left[\sum_{i=1}^{n}\left[(y_i - \hat{y}_i)/\hat{y}_i\right]^2/n\right]^{0.5}$$

= relative root mean square error

where
y_i = Observed value of *i*:th observation
$\hat{y}_i$ = Predicted value of *i*:th observation
n = Number of observations

In general, the tree growth models resulted in a slight underprediction when applied to the test material (Table 9, Fig. 8). Diameter growth model [1] resulted in a smaller average bias than model [3], which included an explicit thinning response variable. Both models showed no biased behaviour as regards thinning intensity (Fig. 8b). However, among the test material, there were only a small number of heavily thinned sample plots; only on three of the sample plots was more than 35 % of the basal area removed in thinning, and only on one plot more than 50 %. Therefore, with the test material as the basis, it could not be reliably confirmed how necessary it would be to incorporate an explicit thinning response variable in the diameter growth model when predicting the development of heavily thinned stands.

The residual mean square errors of diameter growth models [1] and [3] in the test data were notably greater compared to those in the modelling data. This was probably due to larger overall variation in the test data compared to the modelling data. In the test data, the average five-year diameter growth was 1.278 cm with a standard deviation of 0.809 cm. In the modelling data, the corresponding values were 1.147 and 0.546 cm, respectively.

The validation of the height growth model confirmed that the applied model structure is feasible in height growth prediction in both thinned and unthinned stands (Fig. 9b). The model seemed to underpredict the height growth of trees with slow predicted height growth (Fig 9a). However, further data analysis showed that all the observations of trees with predicted height growths of less than 1.2 m were obtained from the one experimental stand only.

For validation of form factor model [8], the form factors for all the trees in the test material were first predicted using model [8], separately for every measurement instance. Thereafter, the stem volumes were calculated using formula [6]. Finally, the predicted stem volumes were compared with the stem volumes calculated on the basis of field measurements.

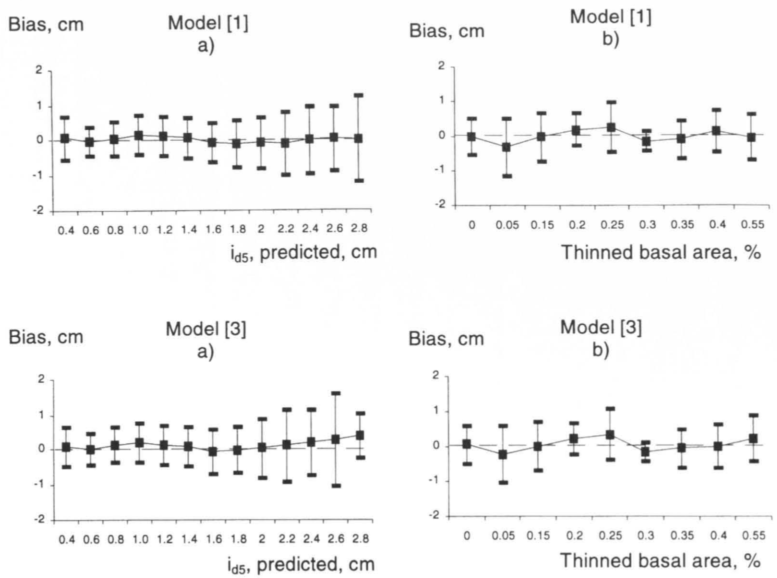

Fig. 8. Average bias (± standard deviation of the residuals) of the diameter growth models [1] and [3] in the test data plotted against predicted tree diameter growth (a), and thinning intensity (b).

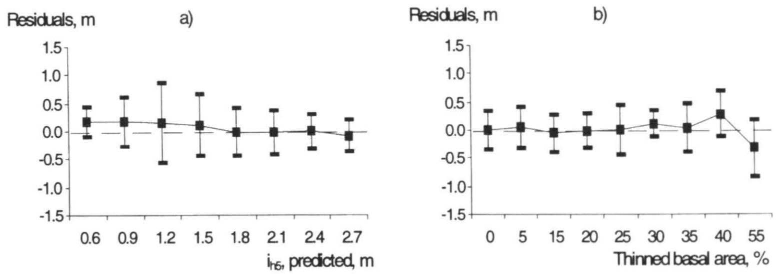

Fig. 9. Average bias (± standard deviation of the residuals) of the height growth model [5] in the test data plotted against predicted tree height growth (a), and thinning intensity (b).

In general, tree volume prediction resulted in a 5.8 % overprediction (Table 9), but there were no trends to be seen in the model prediction regarding thinning intensity (Fig 10b). Nevertheless, there was a slight trend as regards tree diameter; overprediction was at its maximum among the smallest trees (Fig. 10a).

In assessing the results concerning the reliability of stem volume prediction, it must be observed that stem volumes in the modelling data were calculated using simultaneous equations based on tree diameter observations at absolute and relative heights along the stem. In the test data, a three-parameter volume equation ($v = f(d, d_{6.0}, h)$) was applied.

The effect of the applied equation on volume

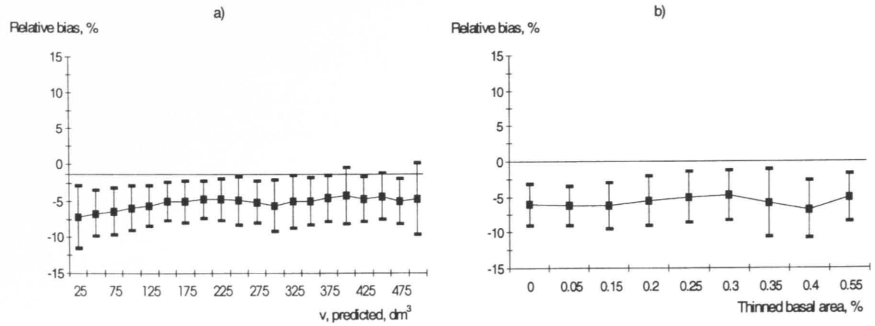

Fig. 10. Average relative bias (± standard deviation of the residuals) of the predicted stem volume calculated with models [6] and [8] in the test data plotted against predicted stem volume (a), and thinning intensity (b).

calculation was tested using the modelling data, in which tree volumes were also calculated using the three-parameter volume equation, i.e. with the same equation as was used in the test data. It was observed that the simultaneous equation resulted, on average, in a 3.5 % greater stem volume compared to the volume estimate calculated using the three-parameter volume equation. Because form factor model [8] is based on the data in which volumes were calculated using the simultaneous equations, it can be said that most of the bias (Table 9, Fig. 10) can be explained by the differences in the volume estimates obtained using these two different volume equations.

6.2 Model Reliability in Predicting Stand Basal Area and Volume Increment

The reliability of the models in predicting stand-level characteristics was examined by simulating the stand basal area and volume increment. Mortality was taken into account in the simulations by removing from the list the trees that had died during the simulation period, before any comparisons were made. The tree volumes at the beginning of the study period were estimated for all trees in the test data using formula [6], in which the tree form factor was calculated using model [8]. Tree diameter increment was simulated using models [1] or [3], and tree height growth using model [5]. In the height growth prediction, the observed dominant height increments (IH_{dom}) on the sample plots were employed as the height growth potential. At the end of the five-year simulation period, the tree volumes were recalculated. Volume growth was calculated as the difference between tree volume at the end and at the beginning of the simulation period. Total volumes of the growing stock and of the stand basal areas of the sample plots were obtained by summing up the tree volumes and tree basal areas, respectively.

Underprediction of the stand basal area increment was obtained with both diameter growth models (Table 10). The model behaviour was similar to that of the tree diameter growth model (Table 9). In stand volume prediction the bias was smaller, mainly because the underestimation of the basal area increment was offset by the overprediction of tree volumes. No dependence between bias and thinning intensity could be observed in basal area or in volume increments (Figs. 11 and 12).

Table 10. Statistics describing the reliability of the model in the prediction of stand basal area and volume increment of the test data during the 5-year growth period.

	Basal area increment		Volume increment	
	Model [1]	Model [3]	Model [1]	Model [3]
Observed	3.495 m^2 ha^{-1}	3.495 cm	42.40 m^3 ha^{-1}	42.40 m^3 ha^{-1}
Predicted	3.347 m^2 ha^{-1}	3.174 cm	43.38 m^3 ha^{-1}	42.08 m^3 ha^{-1}
Absolute bias	0.148 m^2 ha^{-1}	0.320 cm	–0.985 m^3 ha^{-1}	0.315 m^3 ha^{-1}
Relative bias	0.068	0.143	–0.009	0.038
RMSE	0.733	0.796	6.297	6.355
$RMSE_r$	0.303	0.351	0.154	0.178
Observations	28	28	28	28

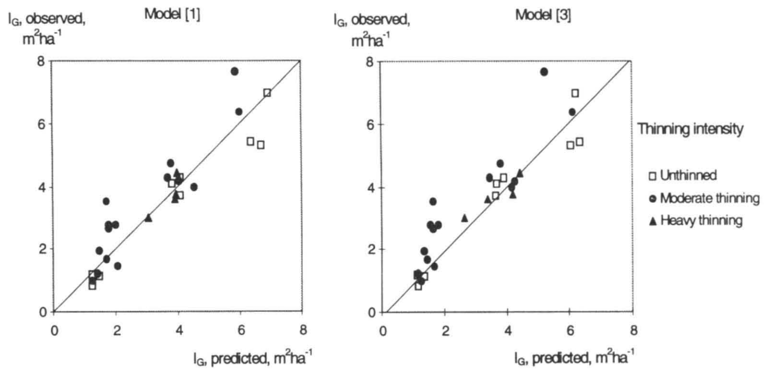

Fig. 11. Observed and predicted stand basal area increments with varying thinning intensity in the sample plots of the test data. In moderate thinning < 30 %, and in heavy thinning > 30 % of stand basal area was removed.

7 Discussion

The main purposes of this study were to examine modelling methods used in predicting the growth response of Scots pine to thinning, and to develop a model structure that can be applied in growth simulations to thinned, as well as unthinned stands. Both the modelling data and the test data were obtained from intensively managed experimental stands. Therefore, the models are not directly applicable to the average commercial Scots pine stands in Finland. Nevertheless, the description of the interactions between growth factors, as well as the basic model structures developed in this study, are likely to be valid also when applied to more comprehensive and more representative data.

Due to the hierarchical data structure, there were both temporal and spatial correlations among the observations. Applying OLS estimation with this kind of data does not cause bias in the parameter estimates, but the standard errors of the parameter estimates are likely to be too small. The hierarchical data structure with the

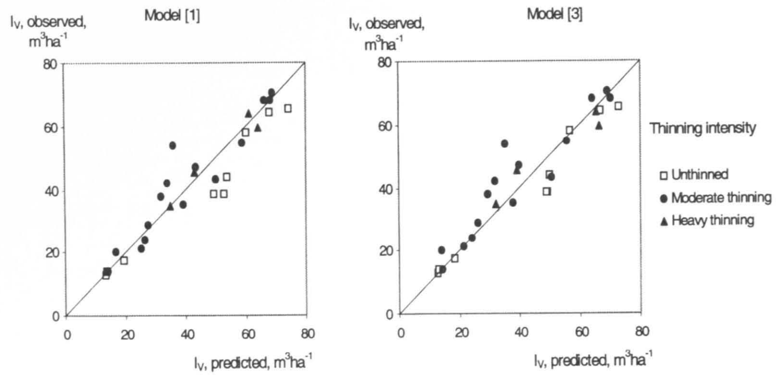

Fig. 12. Observed and predicted stand volume increments with varying thinning intensity in the sample plots of the test data. In moderate thinning, < 30 %, and in heavy thinning > 30 % of stand basal area was removed.

correlated observations can be taken into account in parameter estimation by applying the Generalized Least Squares (GLS) estimation. However, the main interest in this study was in examining the behaviour of and possible biases in the growth prediction obtained when using alternative growth models. Since model predictions are unbiased even when applying the OLS estimation, the main results obtained from the analyses were not influenced by the applied parameter estimation method. Furthermore, because most of the models were intrinsically nonlinear, fitting nonlinear models with the GLS estimation would have been statistically extremely complicated.

A diameter growth model was developed in which the effect of thinning was predicted using an explicit variable referring to the time and intensity of thinning. The analysis confirmed the suitability of the Weibull function in predicting the temporal variation of the growth response. A similar model structure has been employed previously in predicting growth response to nitrogen fertilization (Hynynen 1993). Model [3] proved to be capable of predicting the dynamics of the thinning response, and to perform satisfactorily in both unthinned and thinned stands. In accordance with model [3], the relative growth response to thinning was assumed not to be affected by tree size or by any stand-level variables. The residuals against the modelling and test data indicated unbiased model behaviour with respect to these variables, thus verifying the assumption to be valid in the data sets used in this study. The results comply with the earlier findings of Moore et al. (1994).

In general, both diameter growth models [1] and [3] resulted in unbiased prediction when applying varying thinning intensities. Only in heavily thinned stands did model [1] result in underprediction. Further examination of growth during successive 5-year growth periods revealed that model [1] failed to predict the dynamics of post-thinning diameter growth (Fig. 2b). During the first post-thinning 5-year period, model [1] overpredicted diameter growth, thus indicating that a reduction in the stand basal area through thinning did not immediately increase growth as much as was predicted by model [1]. Correspondingly, during the 5–10 years after thinning, when the thinning response was at its maximum (Fig. 3), model [1] resulted in notable underprediction of diameter growth, especially in heavily thinned stands (Fig. 2b).

The results obtained verified that an explicit thinning variable in a growth model is needed to reliably predict the dynamics of post-thinning diameter growth and to reliably predict tree growth in heavily thinned stands (with more than 50 % of the basal area removed). However, there are some disadvantages in incorporating an explicit thinning variable in the model. First, including an explicit thinning variable in a growth model is likely to result in a rather complicated model structure. Second, the model will require detailed information about thinnings in order to be capable of predicting the growth response.

The diameter growth model [1] without any explicit thinning variable performed adequately in unthinned and moderately thinned stands. It is likely to be sufficient for most practical applications, because so far in practical forestry in Finland thinning intensities have rarely exceeded 50 % of the stand basal area. Model [1] does not require any information about thinning, and this is an important advantage considering the practical application of the model. Information about the timing and intensity of thinnings is seldom available in forest inventory data. In regard to parameter estimation, model [1] can be linearised by using logarithmic transformation, following which the parameters can be estimated with linear regression, and also by applying GLS estimation. In this study, model [1] was not linearised, because that would have complicated the comparisons with intrinsically nonlinear model [3].

Tree crown ratio has been widely applied in growth and yield models as the major driving variable (e.g. Belcher et al. 1982, Burkhart et al. 1987). It is the only variable measured in forest inventories that is directly related to the size of the photosynthetically active biomass. The efficiency of the tree crown ratio as a regressor in diameter and height growth models for Scots pine was tested in this study. Although the crown ratio is known to be correlated with variables referring to stand density (stand basal area), it proved to have a great impact on diameter growth prediction. Including the tree crown ratio in models [1] and [3] significantly improved their accuracy. However, it did not have any major effect on the behaviour of the model with respect to thinning intensity. In height growth prediction, the crown ratio did not prove to be a significant regressor in the present data.

In the model for tree height growth [5], a widely applied model structure was employed, and it proved to be suitable also for Scots pine stands. Height growth prediction in the case of an individual tree was bound to the stand dominant height increment, which was used as the potential height growth. In applying the model, dominant height increment can be obtained from site index equations (e.g. Vuokila and Väliaho 1980, Gustavsen 1980). Therefore, the final performance of the height growth model, when applied in practice, depends on the performance of the site index equation employed in the prediction of dominant height increment. The modelling data of this study were not comprehensive enough to be used in the development of new site index equations.

The dominant height increment was not affected by thinning from below. This is a result that markedly supported the findings of earlier studies (e.g. Hägglund 1974, Vuokila and Väliaho 1980). According to the results of this study, the height growth of an individual tree in a thinned stand can be adequately predicted without any explicit variable referring to thinning.

The aim of the analysis on tree stem form was to examine whether the well-known change in stem form following thinning can be explained only by the change in the *d*/*h* ratio. A relatively simple model for the cylindrical form factor was developed (model [8]) based only on the information about tree diameter and height. Adding other tree and stand variables as regressors into the model improved the model performance only slightly. The model residuals, as well as the validation against an independent data, revealed no biased behaviour with respect to thinning intensity. This being the case, it is reasonable to conclude that, at least in these data sets, there was no change in stem form that could not be explained by change in the *d*/*h* ratio.

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References

Arney, J. D. 1985. A modeling strategy for the growth projection of managed stands. Canadian Journal of Forest Research 15: 511–518.

Assmann, E. 1970. The principles of forest yield study. Pergamon Press. Oxford. 506 p.

Baule, B. 1917. Zu Mitscherliches Gesetz der physiologischen Beziehungen. Landw. Jahrbuch 51: 363–385.

Belcher, D.W., Holdaway, M.R., & Brand, G.R. 1982. A description of STEMS. The Stand and Tree Evaluation and Modeling System. US Forest Service, North Central Forest Experiment Station, General Technical Report NC-79. 18 p.

Burkhart, H. E., Farrar, K.D., Amateis, R.L., & Daniels, R. F. 1987. Simulation of individual tree growth and stand development in loblolly pine plantations on cutover, site-prepared areas. School of Forestry and Wildlife Resources, Virginia Polytechnic Institute and State University, Blacksburg, Publication FWS-1-87. 47 p.

Cajander, A.K. 1909. Über Waldtypen. Acta Forestalia Fennica 1. 175 p. [In German].

Clutter, J.L., Fortson, J.C., Pienaar, L.V., Brister, G.H., & Bailey, R.L. 1983. Timber management: a quantitative approach. John Wiley & Sons. 333 p.

Daniels, R.F. & Burkhart, H.E. 1975. Simulation of individual tree growth and stand development in managed loblolly pine plantations. School of Forestry and Wildlife Resources, Virginia Polytechnic Institute and State University, Blacksburg, Publication FWS-5-75.

Dell, T.R., Feduccia, D.P., Campbell, T.E., Mann, W.F. & Polmer, B.H. 1979. Yields of unthinned slash pine plantations on cutover sites in the West Gulf Region. USDA Forest Service, Research Paper SO-147.

Dyer, M. E. & Burkhart, H. E. 1987. Compatible crown ratio and crown height models. Canadian Journal of Forest Research 17: 572–574.

Ek, A.R. & Monserud, R.A. 1975. Methodology for modelling forest stand dynamics. School of Natural Resources, University of Wisconsin, Madison Staff Paper Series 2.

Gustavsen, H.G. 1980. Talousmetsien kasvupaikkaluokittelu valtapituuden avulla. Summary: Site index equations for conifer stands in Finland. Folia Forestalia 454. 31 p.

Hägglund, B. 1974. Övre höjdens utveckling i tallbestånd. Skogshögskolan, Institutionen för Skogsproduktion, Rapporter och Uppsatser 31. 54 p.

— 1981. Forecasting growth and yield in established forests. An outline and analysis of the outcome of a subprogram within the HUGIN project. Swedish University of Agricultural Sciences, Department of Forest Survey, Report 31. 145 p.

— , Karlsson, C., Remröd, J, & Sirén, G. 1979. Contortatallens production i Sverige och Finland. Projekt HUGIN, Rapport 13. 133 p. [In Swedish].

Harrison, W.C., Burk, T.E. and Beck, D.E. 1986. Individual tree basal area increment and total height equations for Appalachian mixed hardwoods after thinning. Southern Journal of Applied Forestry 10: 99–104.

Heinonen, J. 1994. Koealojen puu- ja puustotunnusten laskentaohjelma KPL. Käyttöohje. Metsäntutkimuslaitoksen tiedonantoja 504. 80 p. [In Finnish].

Hynynen, J. 1993. Modelling tree basal area growth response after nitrogen fertilization. In: Burkhart, H.E., Gregoire, T.G. & Smith, J.L. (eds.). Modelling stand response to silvicultural practices. Proceedings of the IUFRO S4.01 Conference, Blacksburg, Virginia, USA, Sept. 27–Oct 1, 1993. p. 61–72.

— 1995. Predicting tree crown ratio for unthinned and thinned Scots pine stands. Canadian Journal of Forest Research 25. In press.

Jonsson, B. 1969. Studier över den av väderleken orsakade variationen I årsringsbredderna hos tall och gran i Sverige. Summary: studies of variation in widths of annual rings in Scots pine and Norway spruce stands due to weather conditions in Sweden. Skogshögskolan, Insitutionen för skogsproduktion, Rapporter och Uppsatser 16. 297 p.

— 1974. The thinning response of Scots pine (Pinus silvestris) in Northern Sweden. Royal College of Forestry, Department of Forest Yield Research, Research Notes 28. 41 p.

Laasasenaho, J. 1982. Taper curve and volume functions for pine, spruce and birch. Communicationes Instituti Forestalis Fenniae 108. 74 p.
Leary, R.A. 1979. Design. In: A generalized forest growth projection system applied to the Lake States Region. USDA Forest Service, General Technical Report NC- 49: 5–15.
Mielikäinen, K. & Timonen, M. 1995. Männyn ja kuusen kasvunvaihtelu Suomessa 1963–1993. Manuscript. [In Finnish].
Moore, J.A., Zhang, L., & Newberry, J.D. 1994. Effects of intermediate silvicultural treatments on the distribution of within-stand growth. Canadian Journal of Forest Research 24: 398–404.
Näslund, M. 1937. Skogsförsöksanstaltens gallringsförsök i tallskog. Zusammenfassung: Die Durchforstungsversuche Forstlichen Versuchsanstalt Schwedens in Kiefernwald. Meddelanden från Statens Skogsförsöksanstalt 29(1). 169 p. [In Swedish, summary in German].
Ojansuu, R., Hynynen, J., Koivunen, J. & Luoma, P. 1991. Luonnonprosessit metsälaskelmassa (MELA) – Metsä 2000 -versio. Metsäntutkimuslaitoksen tiedonantoja 385. 59 p. [In Finnish].
SAS Institute Inc., 1989. SAS/STAT User's guide. Version 6, 4th ed. SAS Institute Inc., Gary, NC. 846 p.
Shafii, B., Moore, J. A. & Newberry, J. D. 1990. Individual-tree diameter growth models for quantifying within-stand response to nitrogen fertilization. Canadian Journal of Forest Research 20: 1149–1155.
Short, E. A. III & Burkhart, H. E. 1992. Predicting crown-height increment for thinned and unthinned loblolly pine plantations. Forest Science 38: 594–610.
Söderberg, U. 1986. Funktioner för skogliga produktionsprognoser – Tillväxt och formhöjd för enskilda träd av inhemska trädslag i Sverige. Functions for forecasting of timber yields – Increment and form height for individual trees of native species in Sweden. Swedish University of Agricultural Sciences, Section of Forest Mensuration and Management, Report 14. 251 p.
Valinger, E. 1990. Inverkan av gallring, gödsling, vind och trädstorlek på tallarnas utveckling. Influence of thinning, fertilization, wind and tree size on the development of Scots pine trees. Sveriges lantbruksuniversitet, Institutionen för skogsskötsel. Avhandling. 132 p.
Vuokila, Y. 1960. Männyn kasvusta ja sen vaihtelusta harventaen käsitellyissä ja luonnontilaisissa metsiköissä. Summary: On growth and its variations in thinned and unthinned Scots pine stands. Communicationes Instituti Forestalis Fenniae 52(7). 38 p.
— 1987. Puuntuotoksen tutkimussuunnan kestokokeiden periaatteita ja suunnitelmia. Metsäntutkimuslaitoksen tiedonantoja 239. 229 p. [In Finnish].
— & Väliaho, H. 1980. Viljeltyjen havumetsiköiden kasvatusmallit. Summary: Growth and yield models for conifer cultures in Finland. Communicationes Instituti Forestalis Fenniae 99(2). 271 p.
Wykoff, W.R., Crookston, N. L. & Stage, A. R. 1982. User's guide to the Stand Prognosis model. USDA Forest Service. General Technical Report INT-133.

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