A Method for Generating Stand Structures Using Gibbs Marked Point Process

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Stand growth modelling based on single tree responses to their surroundings requires a description of the spatial structure of a stand. While such detailed information is rarely available from field measurements, a method to create it from more general stand variables is needed.

A marked Gibbs point potential theory combined with Markov chain Monte Carlo (MCMC) random process was used to create a spatial configuration for any given number of trees. The trees are considered as charges rejecting each other and building 'potential energy'. As an analogue of the potential energy in physical systems, the potential of a stand is defined in terms of size-dependent tree-to-tree interactions that can be thought of as related to resource depletion and competition. The idea that bigger trees induce larger potentials brings 3-dimensional effects into the system. Any feasible spatial structure is a state of the system, and the related potential can be calculated. The probability that a certain state occurs is assumed to be a decreasing function of its potential. Because more regular structures have lower potentials, by adjusting the steepness of the probability distribution the spatial structure can be allowed to have a lot of randomness (naturally regenerated stands) or forced to be very regular (planted stands). The MCMC algorithm is a numerical method of finding stand configurations that correspond to the expected level of the potential, given the size distribution of trees and the shape of the probability density function.

The method also allows us to take into account spatial variation in the terrain. Some spots can be defined to have lower basic potential than others (ditch, planting furrow, etc.) in order to create areas of higher than average stocking density. A preliminary test of the method was conducted on two measured stands. The results suggest that the method could provide an efficient and flexible means of mimicking variable stand structures.

Keywords stand simulation, spatial distribution, Gibbs point process, Markov chain Monte Carlo

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1 Introduction

The rapid development of computer software recently has increased the number of growth models that treat the stand as a spatially explicit structure, with individual trees interacting with each other through their local environment (Pretzch 1992, Courbaud 1995, Pacala et al. 1993, Williams 1996). In order to initialise and test the projections of such models, adequate descriptions of the spatial distribution of trees in a stand are needed (Pukkala 1988b). In most cases, the spatial information required by the models is not available, or is hard to come by using empirical measurements. Instead, mathematical estimation procedures have to be employed that can be tied with measurements through some aggregate characteristics of the set of trees considered.

The most common method to create a two-dimensional point pattern for tree locations is provided by the Poisson process (e.g. Valentine et al. 1999). Some inhomogeneity can be reached by dividing the area considered into subareas and drawing a random number from the Poisson distribution for each subarea to represent the number of trees located in this area. The distribution can be adjusted according to mean density and the number of subareas used. Even some clustering effects can be created by adding another parameter into the algorithm (Penridge 1986).

Two problems arise in applications of the Poisson algorithm to tree stands. Firstly, the points are independent of each other, discarding any possible effects of crowding. Secondly, no marks are attached to the points, whereby the effects of tree interactions cannot be taken into account. However, in reality both locations and trees have additional characteristics, such as tree size or local soil moisture, which have mutual correlations that affect the spatial structure (Penttinen et al. 1992). In order to include the interactions between tree size and location, Valentine et al. (1999) modified the Poisson algorithm to account for the fact that large trees would normally occupy a larger area than smaller trees. They generated a Poisson point pattern, but instead of randomly locating trees from a given size

distribution to these points, they ordered the trees according to size and placed the larger trees to the points where neighbours were furthest.

Although incorporating some effects of competition, this modification does not dispense with the problem of independent point pattern. That could only be taken into account in a so-called marked point process (e.g. Snyder 1975, Penttinen et al. 1992), assigning trees and locations additional characteristics that may interact with each other, modifying the distribution of tree locations also. The theory of marked point processes has been applied to tree stands and to describe statistical features in them (Mateu et al. 1998). Methods for generating stand structures using such algorithms have recently been reviewed by Stoyan and Penttinen (2000).

Gibbs pair potential field theory (e.g. Thompson1972, Ripley 1988, Bar-Yam 1997) provides an attractive analogue for a method for placing trees in a stand. The theory finds a spatial distribution for objects that are assumed to interact pairwise, the interaction between each pair of objects acting like a repulsive force that pushes the objects away from each other. According to Gibbs' theory, the probability of a particular configuration of objects is the greater, the lower the total potential in the system. However, the expected configuration is not the absolute potential minimum (which in the case of identical trees on a uniform site would be a regular grid) but a state where low potential balances with random movement of objects. In the analogue of the theory to trees in a stand, the "repulsive force" is defined as competition between the trees, and the "random movement" is analogue with the randomness found in the regeneration process. Local site variation can be included as additional sinks and sources of the "potential field". When the probability distribution of the possible states is simulated, configurations giving rise to the expectation value of the potential can be found. These represent a balance between the minimum potential and the hypothesized degree of randomness.

However, although this method would appear promising in principle, attempts to apply the Gibbs potential field analogue to the generation of stand structures so far have provided inconclusive results. Ogata and Tanemura (1985) parameterised a piecewise continuous potential function

from measurements, but the application of this function failed to reproduce the measured stand characteristics (Goulard et al. 1996). On the other hand, Stoyan and Penttinen (2000) showed that a fairly regular stand structure could be simulated using pair potentials defined on the basis of mutual distance only. These experiences suggest that the critical features of the method for realistic simulations, are the definitions of the pair potential and the background potential field, as well as the relative weights of the two. The pair potential is responsible for the repulsive effects of competition between trees, while the background potential field can create the opposite effect of clustering.

In this paper, we apply the Gibbs marked point process to generate the spatial structure of a stand of saplings, given their diameter distribution. Special attention is paid to the definition of the pair potential and the background potential functions, using ideas of tree-to-tree competition and ecological field theory. The properties of the method are reviewed, with special attention to the possibilities of estimating the parameters of the method consistently for specified stand types. A preliminary comparison with measured stand structures is presented, with the objective of assessing the applicability of the method as an initialising algorithm for stand growth models.

2 Material and Methods

2.1 Gibbs Marked Point Process

2.1.1 Principle of the Method

On a marked Gibbs point potential field there are a number of points on a given area. The points are marked, i.e. their values differ from each others. These values determine the potential field they create around themselves. While points keep pushing (at least some of the potentials must be repulsive, otherwise the problem would be trivial) each other away, they can never leave the area, so they try to arrange themselves, so as to minimise the overall potential. On the other hand, there is always a random factor in the process causing transitions sometimes to take place upwards in

the potential.

Calculating the total potential of any pattern is simple, but since there are an infinite number of possible patterns, it is not possible to find the one with the expected potential in closed form. Instead, we define the possible patterns as states in a Markov chain and apply the Metropolis-Hastings simulation (e.g. Ripley 1981, Penttinen 1983, Bar-Yam 1997, Dam et al. 1999) on it. All patterns are arranged according to their potential. By moving one point at a time to a new location and following the change in total potential at every step, we can study the behaviour of the system and the probabilities of its states.

After a large number of random steps, the potentials of the generated Markov chain approximate the probability distribution of the potential. This allows us to estimate the mean potential, and further, to select any configuration giving rise to a potential close enough to the mean. In the following, the assumptions concerning the potential field, and the steps of the procedure are explained in more detail.

2.1.2 Description of the Potential Field

Let N be the number of trees in the stand and A the stand area ($A \subset R^2$). For convenience, we define A as a rectangle. Every tree has a location in A, which is denoted by a vector x ($x \in A$). Let $X = X(x_1 x_2 \ldots x_N)$ be a vector ($X \in A^N$) including all horizontal locations of the trees of the stand. X is a spatial structure of the stand.

In order to define the Gibbs potential field for the trees in the stand, we need to define a pair potential, Φ_{ij}, between any two trees i and j. We hypothesize that the pair potential quantifies the effect of competition between trees on the spatial structure. It is therefore close to the concept of the competition index, often used in growth modelling. A crucial difference is, however, that the potential is defined for a pair of trees, not for each individual, and therefore has to be symmetrical, i.e., $\Phi_{ij} = \Phi_{ji}$. This is not true of most competition indices which depend on the relative sizes of the competing trees.

Competition is defined as interaction between trees due to the utilisation of shared resources (Ford and Sorrensen 1992, Vanclay 1994). The

resources are drawn from a zone of influence around the tree, which is the larger, the bigger the tree. The "repulsive force" between any two trees should therefore increase (or remain constant) with the size of the trees. On the other hand, because the intensity of utilisation of resources decreases with the distance from the tree, the pair potential should decrease (or remain constant) with increasing mutual distance.

There are several possible formulations satisfying the three requirements above. For example, Penttinen et al. (1992) and Goulard et al. (1996) defined piecewise continuous / differentiable functions of mutual distance that decreased with increasing distance and were zero for distances larger than a given limit, while the functions were independent of tree size. We suggest the following formulation, which satisfies the requirements set above:

$$\phi_{ij} = c \cdot \frac{d_i \cdot d_j}{\left\| x_i - x_j \right\|^q} \tag{1}$$

where d_i is the breast height diameter of tree i and c is an empirical constant which may depend on the species of i and j and soil fertility. In the denominator we have the distance between trees raised to the power q. In the above form the competition potential goes to infinity when the tree-to-tree distance approaches zero and vanishes gradually as the distance increases. The potential is also highly dependent on the tree sizes; the potential becomes very big only in the case of two large trees. Like in reality, one may find a small seedling or a suppressed tree close to a big one, but two big trunks very close to each other are highly unlikely.

Assuming that the area of competition around a tree can be defined as a circle with radius proportional to d_i, and that the intensity of competition is proportional to the overlap of the competition areas of neighbouring trees (Fig. 1), it turns out that Eqn (1) with $q = 1$ approximates this overlap area, the approximation being the more accurate, the more similar the tree sizes. The value $q = 1$ is used in this article. Goulard et al. (1996) also used the overlap area as the basis of the pair potential function. However, if the assumptions do not hold, values up to 2 might be justified. It is also interesting that the above formulation is
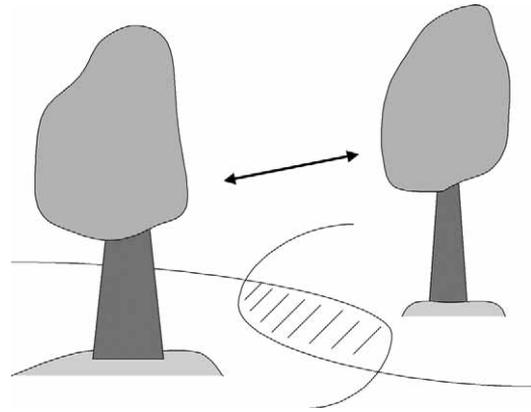

Fig. 1. Competition between a pair of trees depends on the size and distance of the trees. As the distance decreases, the competition (dashed zone) enlarges.
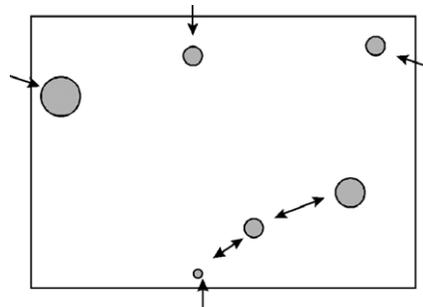

Fig. 2. Periodical boundaries.

similar to the well-known law of gravitation.

Now we can calculate the total competition potential within the stand, U, by summing up all the tree pairs.

$$U_\phi(X) = \sum_{i<j} \phi_{ij} \tag{2}$$

To avoid tricky boundary effects we determine the stand area to be periodic. This means connecting the left and right boundaries as well as the upper and lower (Fig. 2.) Mathematically our stand surface is similar to a torus.

The method of calculating competition potentials can be used not just to describe tree-to-tree competition, but we can also take into account various kinds of irregularity in the growth site. For example a moist creek can reduce potential locally so there will be more trees than average.

Vice versa, we can use increased competition potential on some rough (e.g. rocky) plots. The magnitude of the site effect on a tree is usually related to the size of the tree.

Let $s = s_i(x_i)$ be the site effect on tree i in location x_i.

$$U_s(X) = \sum_i s_i(x_i) \tag{3}$$

The shape of s may depend on the species of tree i. In the examples of this article, though, the function s is assumed to be similar for all species so, s_i can simply be written as s.

Combining (2) and (3) we get the total potential

$$U(X_k) = \sum_{i<j} \phi(d_i, d_j, \|x_i - x_j\|) + \sum_{i=1}^{N} s(x_i) \tag{4}$$

Using Gibbs' theory we can calculate the total competition potential for every structure.

2.1.3 Probability Distribution of Structures

According to Gibbs' theory, the probability, $f(X)$, of each structure, X, can be calculated from the formula (Thompson 1972)

$$f(X) = \frac{1}{Z} \cdot e^{-\beta U(X)} \tag{5}$$

where Z is a scaling factor and $\quad$ is a parameter determining the degree of randomness. In order for $f(X)$ to be a probability distribution all terms must sum up to unity. Thus we can determine Z as

$$Z = \int_{X \in A^N} e^{-\beta U(X)} dX \tag{6}$$

The expectation value of $U(X)$, i.e. the expected potential of the spatial structures with the probability density function given by Eqn (5), is defined as (Ripley 1988)

$$U_{mean} = \frac{1}{Z} \int_{X \in A^N} U(X) e^{-\beta U(X)} dX \tag{7}$$

Since the probability of structure X is totally determined by the related potential, any structure X with this potential is equally likely. Any configuration X giving rise to the mean potential, U_{mean}, can therefore be considered a feasible solution. Because of the enormous number of terms to be calculated, the integral of Eqn (7) cannot be determined analytically. To find configurations with potentials close enough to U_{mean}, we employ the so-called Metropolis-Hastings simulation (Dam et al. 1999).

2.1.4 Steady State of the Markov Chain

By simulating the chain of states for long enough we generate an estimate of the probability distribution of potentials. From this simulated distribution we can determine an estimate of the mean potential. Any configuration with a potential sufficiently close to the estimated mean can be accepted. In order to find a feasible configuration, the following steps were taken:

1) The change of potential is monitored at every step, i.e., at each movement of a tree from one place to another. When changes up and down the potential balance over a chosen number of steps, we assume there is no more systematic drift and the simulated states represent the probability distribution of the potential.

2) The algorithm is run for a large number of steps and the potential minimum, average, and maximum are updated at each step and stored.

3) A configuration is defined to be feasible if its potential is in the range of [estimated mean potential ± 0.02 * (maximum – minimum)]. The next time the potential falls within the accepted range) the algorithm is stopped and the configuration reached is selected.

2.2 Material

Preliminary test material was collected from two plots of saplings in Hyytiälä (61°48', 24°19'), southern Finland. Both plots (outlined inside a larger stand area) were 20 m × 20 m in size, Scots pine (*Pinus sylvestris*) dominated, but included also trees of various other species (Table 1).

In all seedlings at least 10% of the average height, tree height, height of crown base, diameter at crown base, and breast height diameter (when

Table 1. Charcteristics of measured plots.

	Plot 1	Plot 2
Year of establish-ment	Clear felling in 1985	1987
Age	<15	
Method of establishment	Natural regenera-tion with seed trees (removal in 1993), Soil preparation	Planting
Mean height (m)	1.6	5.4
Density (ha^{-1})	11 000	4400
Site type[*]	*Vaccinium* (VT)	*Vaccinium Myrtillus* (MT)

[*] Site type as defined by Cajander (1949)

existent) were recorded. In addition, the location of the seedlings was measured in terms of angular coordinates from a reference point.

2.3 Comparison of Measured and Simulated Stand Configurations

For comparisons of the measured and simulated spatial patterns, we calculated several measures of grouping and spatial correlation. The Hopkins grouping index (e.g. Pukkala 1988b), *g*, measures the degree of clustering, and is defined as

$$g = \sum a_i^2 / \sum b_i^2 \qquad (8)$$

where a_i is the distance from a random point to the closest tree and b_i is the distance from a random tree to its closest neighbour. The smaller the index, the more regular the pattern. For a Poisson process $g = 1$, while $g < 1$ is considered regular and $g > 1$ indicates clustering. Since equation (8) includes a random factor the calculation gives a slightly different value for *g* every time. The variations are small, however, so we simply used the average of seven calculations.

The pair correlation function, *g(r)*, is a (relative) measure of the probability that two trees occur at a mutual distance *r*. For a Poisson process, *g(r)*

1. If distances *r* are more likely than random, $g(r) > 1$, indicating clustering. Values $g(r) < 1$ indicate regularity. In a sense, *g(r)* is a spatial
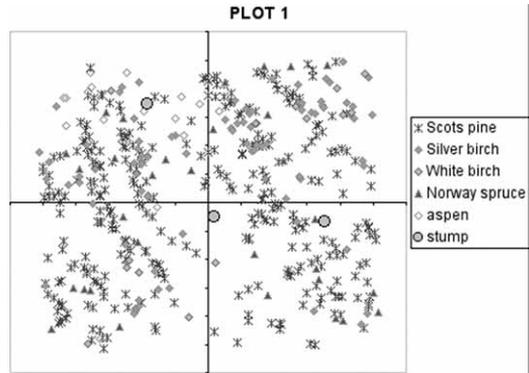

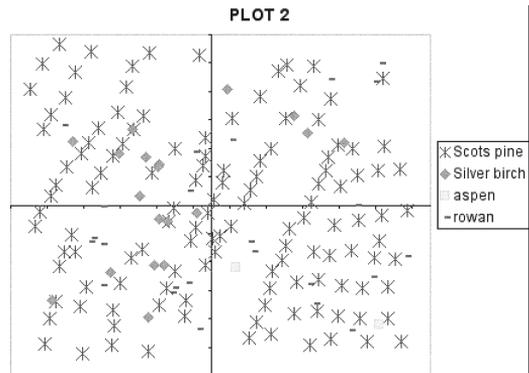**Fig. 3.** Testplot 1.

Fig. 4. Testplot 2.

version of the grouping index defined above. *g(r)* was calculated using the method presented by Penttinen et al. (1992).

The mark correlation function, $_{mm}(r)$, measures the spatial correlation of marks (Penttinen et al. 1992). We calculated $_{mm}(r)$ for tree diameters, *d*. $_{mm}(r) > 1$ for distances *r* where it is likely that tree sizes are similar, and $_{mm}(r) < 1$ for distances at which tree sizes are likely to be dissimilar. For no correlation, $_{mm}(r) = 1$. Again, the numerical estimation was carried out following Penttinen et al. (1992).

The *L* function is essentially a normalised integral of *g(r)*. For a Poisson process, $L(r) = r$. The *L* function can be used as a tool for determining the goodness-of-fit of simulations of spatial point processes (Stoyan and Penttinen 2000). We evaluated a range for the simulated *L* function from nine independent simulations with the same parameter values, and this range was compared with the *L* functions of the measured stands.

3 Application of the Method

3.1 Rate of Convergence

The Markov chain defined above always converges towards the probability distribution $f(X)$ (Eqn 5), however, the rate of convergence depends on various factors. The more trees we have, the longer it takes to reach the equilibrium distribution. Therefore, although the initial structure does not affect the final result, a good guess will accelerate the process.

The rate of convergence also depends on the value of . Three values were compared for an imaginary stand of 124 trees with diameters between 2.4 cm and 4.8 cm, distributed over a plot of 25×25 m^2 in size. The simulation was initialised with an arbitrary (rather grouped) configuration, and the value of the initial potential was used as a reference. It is apparent from Fig. 7 that larger values of   imply lower mean potentials and variance but concurrently slightly slower rates of convergence.

3.2 Adjusting for Regularity

Higher mean potentials correspond to more regular configurations than low mean potentials, and the regularity of the resulting stand structure therefore depends on the value of . The three values of   used above lead to somewhat different stand structures eg. in the sense of Hopkins grouping index. Calculating averages and variances over 11 simulations with each   we received the following. (Table 2)

This property implies that a simple adjustment of   will allow us to choose freely the degree of regularity ($g < 1$) of the stand structure to be cre-

Table 2. Mean Hopkins grouping index (g) and its variance (^{2}) for stand configurations created with different values of   (N = 11).

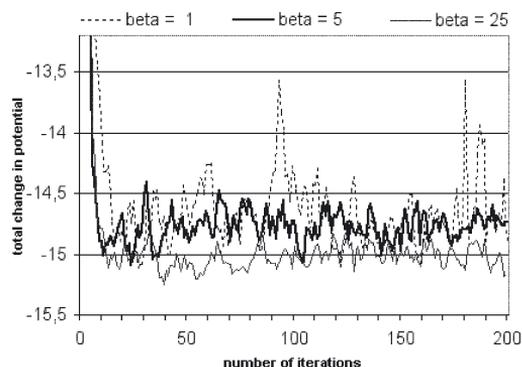|	25	5	1
g	0.82	0.93	0.99
	0.055	0.12	0.14

Fig. 5. Convergence of the simulation method with three different values of . One iteration means 25 move attemps. The y-axis shows the total change in potential from the initial structure.

ated, provided that the desired regularity property can be quantified. In case of very different size of trees we can even receive $g > 1$.

For a given value of , the stands with the larger trees are more regular than those with the smaller trees, since higher total potential allows less randomness. This resembles the development of structure in a real stand. While trees grow competition intensifies especially where density is high. Therefore mortality is highest in dense parts, and as a result the structure becomes more regular.

3.3 Site Effects

To illustrate the effect of site potential on stand configuration, three different cases were simulated. Adding a straight line of lowered potential across the plot caused more trees to group around the line (Fig. 6). Similarly, pointwise decrease in potential attracted trees to the point, while a similar increase in potential created relatively sparse spots in the stand (Fig. 7). A gradient in site potential induced a corresponding gradient in the density of trees accross the plot (Fig. 8).

3.4 Simulating Measured Testplots

An important criterion for the applicability of the present method, the algorithm should be able to
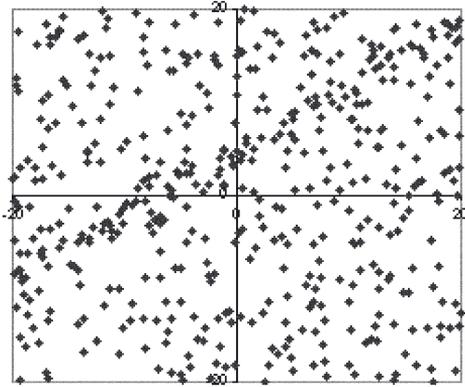
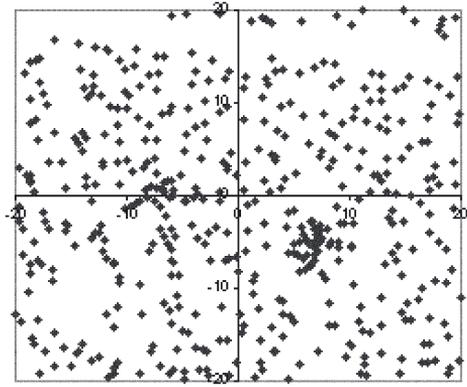**Fig. 6.** A negative site potential across the stand.

Fig. 7. A positive site potential on the low left and up (a forest road). Two negative point potentials close to each other in the middle.
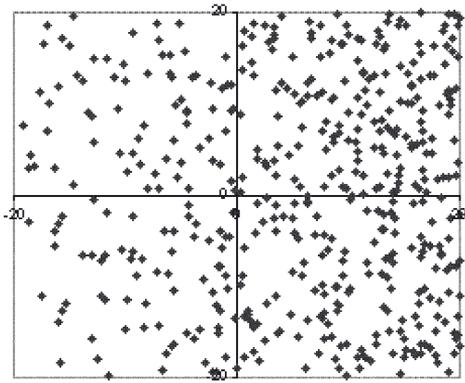

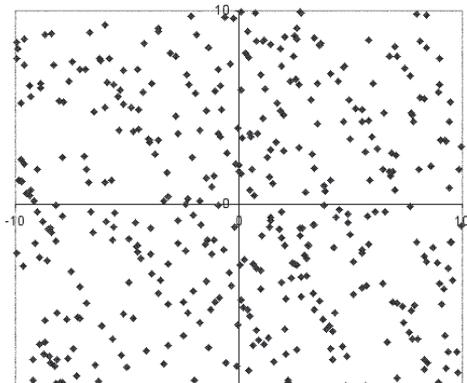**Fig. 8.** Decreasing site potential from left to right.

Fig. 9. Simulated pattern of plot 1. Only planting furrows as site potential.

create stand configurations that resemble actual stands in terms of some measure of regularity. As a preliminary test of this property, we assessed how closely and how consistently the algorithm was able to reproduce the measured stands (Section 2.2). The objective was to simulate the configurations of plots 1 and 2 from input information that could be easily observed from any actual stand.

The algorithm was initialised by placing the measured trees randomly in a periodic 20 m × 20 m rectangle. For both plots, site effects were added to account for the method of regeneration observable in the measured pattern. To simulate the soil preparation in plot 1, we implemented 50-cm-wide strips (the average width of a plough

track) of negative potential at 3 m intervals. The values of c and q (Eq. 1) were set to 15 and 1, respectively. The resulting configuration (Fig. 9) looked rather similar to the original, but the value of the grouping index, 1.1, was much lower than the one calculated for the measured stand (2.1). The pair correlation function, $g(r)$, also seemed too small even at short distances (not shown). To improve the simulation, we inserted the effect of the removed seed trees, measured as stumps on the plot (Fig. 3), into the site potential function as positive terms. This is justified as it seems unlikely for a seedling to be established close to a big seed tree, and the effect is likely to persist even a few years after the removal of the seed tree due to the remaining root system.
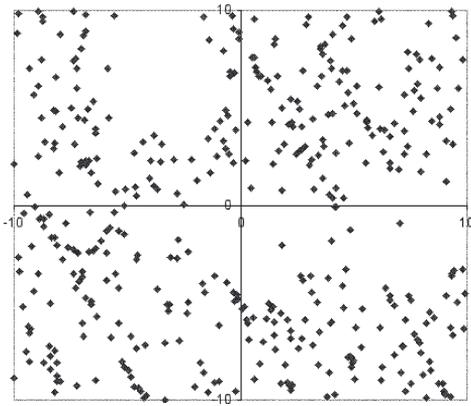
Fig. 10. Simulated pattern of plot 1. Planting furrows and stumps as site potential.

Fig. 11. Pair correlation functions of the measured and simulated patterns of plot 1.

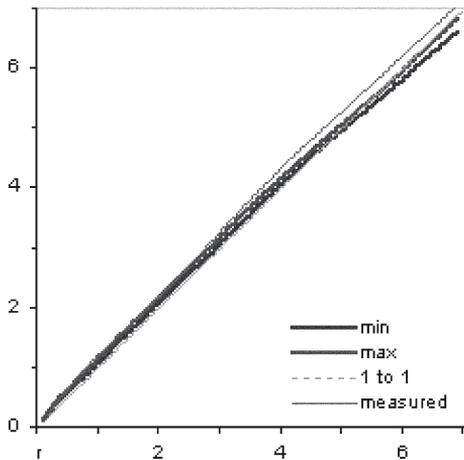**Fig. 13.** Mark correlation functions of plot 1.

Fig. 12. L functions of the measured and simulated patterns of plot 1. The measured pattern is shown as the minimum and maximum of nine simulated configurations.

The introduction of the unfavourable patches made the rest of the trees pack tighter to the remaining areas (Fig. 10). Both site effects, the furrows and the stumps, can be seen clearly in the tree pattern, yet there is still a lot of randomness. The grouping index was 1.6 which indicates clear clustering compared to the Poisson process. The pair correlation functions of both the original and simulated patterns (Fig. 11) peak at very short distances and again around three meters, which is the harrowing furrow interval. In both cases, the
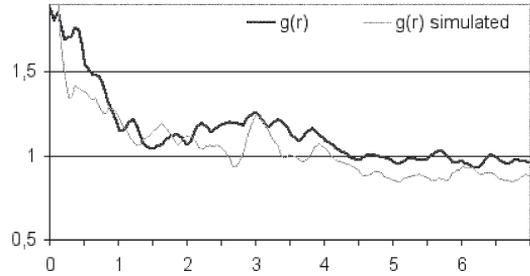
correlation stays positive over the second peak. This suggests that the soil properties have been the dominating factor over tree-to-tree interaction determining the locations of the new seedlings. This is a typical feature of young naturally regenerated stands. Why the pair correlation stays high longer in the measured pattern is probably due to the fact that, while in simulation the harrowing track was a straight line, in reality it has wider and narrower spots which further increase the grouping tendency. This tendency is further manifested by the *L*-function, where the measured *L* stays above the simulated values (Fig. 12).

The mark correlation functions, $k_{mm}(r)$, for measured and simulated stands show similar peaks at distances $r < 1$ m, but after that the simulated function drops below 1 while the measured stays above 1 up to about $r = 3$ m (Fig. 13). For distances $r > 3$, both settle around the value 1 with random fluctuations.

A slightly different approach was selected when simulating the tree pattern of plot 2. Also
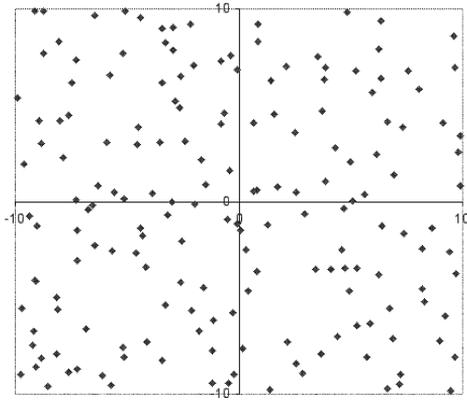
Fig. 14. Simulated configuration of plot 2.

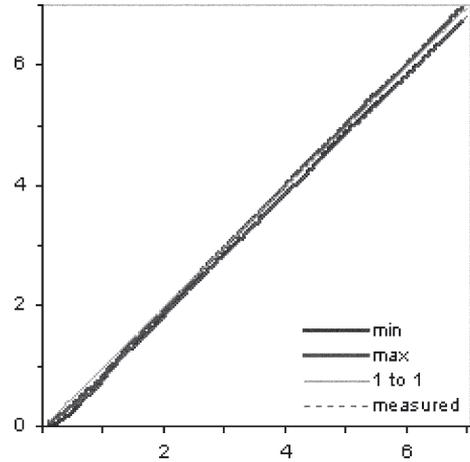**Fig. 15.** Pair correlation functions of the measured and simulated patterns of plot 2

Fig. 16. L functions of the measured and simulated patterns of plot 2. The measured pattern is shown as the minimum and maximum of nine simulated configurations.
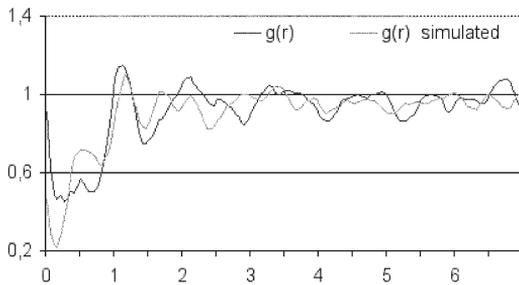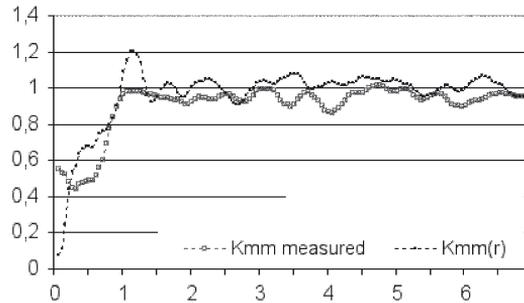

Fig. 17. Mark correlation functions of plot 2.

here the rows of trees are clearly visible and we could employ similar site potential functions as in the previous case. Now, however, since the trees are planted, the dominant factor determining pairwise distances is not the distance between rows but between trees on the same row. We therefore made an artificial correction in the pairwise potential function. If the distance between two trees was between 1.1–1.25 meters (that seems to have been the opted distance between neighbours while planting the seedlings) the tree-to-tree potential term was divided by 10. No site terms were used.

The algorithm now favours tree distances of the given interval (Fig. 14). The grouping index for the simulated pattern is 0.63, as compared with 0.62 for the measured stand. The pair correlation function (Fig. 15), as well as the L function (Fig. 16), are very similar to those of the measured

plot. Since we omitted the site terms, the tree rows in the simulated pattern are not as visible as in the measured plot. This could be corrected as in the case of plot 1 but it would hardly improve the calculated characteristics.

The mark correlation functions K_{mm} are similar for the measured and simulated stands, with $K_{mm}(r) < 1$ for $r < 1$ m, both peaking at $r = 1$m and then settling at about $K_{mm}(r) = 1$ (Fig. 17). The variation in the simulated curve is slightly wider and the equilibrium level slightly higher than the measured, but the qualitatively the two curves are similar.

4 Discussion

We have presented a method to create a 3-dimensional spatial structure for a stand for modelling purposes. As initial information, only tree density and size distribution are needed. The method seems very promising since it is both simple and flexible, but still covers practically any kinds of stand. One equation is enough to give the information needed about the stand structure to the simulation algorithm. Still the algorithm can be adjusted to create regular to random or even clustered patterns. Furthermore, the method can be applied to single species stands as well as mixed stands. Site effects such as inhomogenities in the ground, seeding furrows or seed trees can also be taken into account whenever such information is available. Practically the more we know about the stand we want to create, the more we can include in the algorithm.

The present method differs from previous applications of the Gibbs process to tree stands (Ogata and Tanemura 1985, Goulard et al. 1996) in a number of ways. Perhaps most importantly, we have introduced the effect of site variation as an independent background potential term, $s(x_i)$. This bears some similarity to the Cox process which samples an intensity function for an inhomogeneous Poisson process (Stoyan and Penttinen 2000). The background potential allows for the creation of clustered configurations where the clustering follows the site characteristics, such as effects caused by the regeneration method. To account for similar effects, Goulard et al. (1996) allowed for the pair potential to be attractive at some distances. Secondly, the formulation of the pair potential function differs from previous models which have often been piecewise continous functions of inter-tree distance (Ogata and Tanemura 1985, Stoyan, Kendall and Mecke 1995, Goulard et al. 1996, Penttinen et al. 1992, Stoyan and Penttinen 2000). Thirdly, the present model incorporates quantitative marks in the pair potential function, which has not often been done previously (Stoyan and Penttinen 2000). Ogata and Tanemura (1985) considered qualitative marks in a binary point process, while Goulard et al. (1996) used quantitative marks based on breast height diameter in a piecewise constant model of the pair potential. The present formulation is continuous, includes tree sizes, and can be interpreted as a method of generating a marked point process.

For applications of the present method to stand initialisation in growth models, it is important 1) to test if the generated spatial configurations represent real configurations for some sets of parameter values, and if they do, 2) to define a general method for determining the appropriate sets of parameters for different types of stand. Unlike in other studies on point processes, our objective has not been to estimate the parameters of the process from a given data set, but to try and determine the parameters from some outstanding characteristics of the stand in question, and to test the resemblance between the measured and simulated stand using measures defined for spatial patterns.

Based on the present preliminary test of the method, it seems likely that parameter values can generally be found that reproduce the spatial characteristics of any measured stands. In the highlight of the examples, regular stands may be easier to simulate than clustered stands, but this is the case with any spatial simulation method (Stoyan and Penttinen 2000). In the case of clustered stands, the site potential appears to play a major role. It is encouraging that the measures of spatial correlation generated by the method seem qualitatively consistent with the corresponding measured values, including the mark correlation function $_{mm}$. This suggests that the inclusion of stem diameters in the pair potential function may be adequate. However, larger data sets and further methods of comparison between measured and simulated stands should be used to further test the results. An interesting 3-dimensional tool for comparison could be drawing triangles to tree tops (Zenner and Hibbs 2000). The area of the triangles compared to the ground area measures the stand heterogeneity.

Ultimately, the required degree of accuracy between the simulated and measured stands will be determined by the response of the spatial growth models used. Errors in spatial pattern can be accepted if they have little effect on the overall growth characteristics of the stand. However, another criterion for practical applications is a

visual one: does the simulated structure look like the measured stand? This argument should not be belittled since, especially for a forest professional, growth simulation results are difficult to believe if the initial stage cannot be recognized.

Although the present preliminary analysis has demonstrated that the method bears promise for future applications, a number of problems remain to be solved. Our future work will focus on 1) further tests of the tree-to-tree interaction function and the adequacy of the marked point process defined by it, 2) inclusion of different species in the pair potential function, and 3) examining if more general rules can be constructed for the combination and relative weights of the pair potential function and the site potential, so far treated with 'trial and error'. Most importantly, parameter sets will be defined for generating typical stands of seedlings for simulations in the growth model.

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