

VOLUME AND INCREMENT CALCULATION OF A SAMPLE  
PLOT DETERMINED WITH THE RELASCOPE

SELOSTUS:

KUUTION JA KASVUN LASKENTA RELASKOOPILLA  
MÄÄRITETYLLÄ KOEALALLA

KULLERVO KUUSELA

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## Preface

I have a pleasure to thank professor Aarne Nyssönen who has read the manuscript and given many valuable suggestions concerning the content of this paper, and the Society of Forestry in Finland which has published it.

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*Kullervo Kuusela*

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## Introduction

The relascope method, introduced by BITTERLICH (1948, 1952, 1954 and 1955), has been mostly used this far in estimating the basal area of tree stands or growing stock. Volume estimation requires in addition mean height and form. Empirical tables having tree species, basal area and mean height as variables are thought to give the most accurate results. If the diameter of the trees tallied on a relascope plot is measured, an ordinary stock table can be calculated. Bark and increment have been estimated using bark and increment percentages (NYYSSÖNEN 1954 and 1959, SENDA and MAEZAWA 1955, GOCKERELL 1957, STRAND 1957 and 1957 a and VUOKILA 1959).

The purpose of this study is to work out a method for calculating the volume, bark, and increment of a stand from the measurements of sample trees taken on a plot determined with the relascope. All trees of the same diameter have their own plot size and the stand characteristics are the summation of all tree characteristics multiplied by a blow-up factor which is a function of the diameter (APPELROTH 1956 and GROSENBAUGH 1958).

## Concepts

Every tree tallied on a relascope sample plot represents one unit of the basal area per hectare. If the opening of the relascope is 2 cm. in width and its distance from the eye is 1 m., each tree represents basal area of 1 m<sup>2</sup>/ha. which can be called unit basal area ( $G_u$ ). A tree tallied on the plot is called a unit tree (U). The number of unit trees (N) equals the basal area of the stand under estimation in terms of m<sup>2</sup>/ha. ( $G/\text{ha}$ ):

$$N = G/\text{ha} \quad (1)$$

Distribution of the tallied trees by diameter classes is called the relascope stock table, in which each unit tree is characterized by diameter at breast height (d), basal area (g), height (h), form (f) and volume (v).

One unit tree represents a unit number of trees per hectare ( $N_u$ ) which is calculated by the formula:

$$N_u = \frac{1 \text{ m}^2}{g} \quad (2)$$

The unit numbers can be tabulated as a function of the unit-tree diameter (Table 1). The total number of trees per hectare represented by the unit trees ( $N/\text{ha}$ ) is the summation:

$$N/\text{ha} = \Sigma N_u \quad (3)$$

Unit volume per hectare represented by each unit tree ( $V_u$ ) is calculated by the formula:

$$V_u = N_u \times v \quad (4)$$

The total volume per hectare of the stand under estimation ( $V/\text{ha}$ ) is the summation:

$$V/\text{ha} = \Sigma V_u \quad (5)$$

The arithmetic mean of the unit-tree heights ( $\bar{h}$ ) is needed for volume determination. The mean of the unit basal areas ( $\bar{G}_u$ ), which is smaller than  $1 \text{ m}^2/\text{ha}$ , for basal area excl. bark and for basal area  $n$  years ago excl. bark, and the mean of the unit volumes ( $\bar{V}_u$ ) are calculated as arithmetic means of the corresponding unit characteristics. The total volume per hectare can also be calculated by the following formula:

$$V/\text{ha} = G/\text{ha} \times \bar{V}_u$$

The characteristics described above are considered as including bark (i.b.). The characteristics of the basal area excluding bark (e.b.) are calculated by the formulas:

$$G_u, \text{ e.b.} = N_u \times g, \text{ e.b.} \quad (6)$$

$$G/\text{ha}, \text{ e.b.} = \Sigma G_u, \text{ e.b.} = N \times \bar{G}_u, \text{ e.b.} \quad (7)$$

Unit basal area excl. bark ( $G_{u, \text{ e.b.}}$ ) can be tabulated as a function of the double-bark thickness (d.b.) (Table 2).

Height is the same for a unit tree excl. and incl. bark.

If the diameter increment ( $i_d$ ) of a unit tree during the last  $n$  years is measured and added to the double bark ( $i_d + d.b.$ ), then the unit basal area  $n$  years ago excl. bark ( $G_{u, -n, \text{ e.b.}}$ ) and the corresponding basal area per hectare of the stand under estimation ( $G/\text{ha}, -n, \text{ e.b.}$ ) can be calculated using Table 2 in the same way as in calculating the corresponding characteristics excl. bark.

Height of the unit tree  $n$  years ago ( $\bar{h}, -n$ ) is the present height ( $\bar{h}$ ) minus the height increment ( $i_h$ ) during  $n$  years. The mean height of the unit trees  $n$  years ago ( $\bar{h}, -n$ ) is calculated from the arithmetic mean increment ( $\bar{i}_h$ ) during  $n$  years:

$$\bar{h}, -n = \bar{h} - \bar{i}_h \quad (8)$$

### Principles

Volume incl. bark ( $V/\text{ha}$ ), excl. bark ( $V/\text{ha}, \text{ e.b.}$ ) and  $n$  years ago excl. bark ( $V/\text{ha}, -n, \text{ e.b.}$ ) are the most important objectives of forest mensuration. Volumes

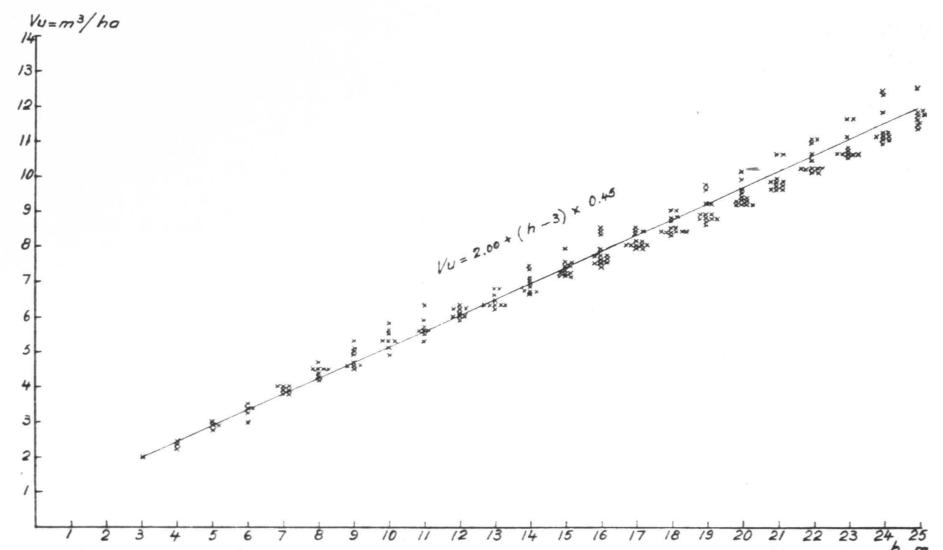


Fig. 1. Unit volumes as a function of the tree height from a volume table with diameter and height as variables.

Piirros 1. Läpimittaan ja pituuteen perustuvan kuutioimistaulukon (Tapion taskukirja) mukaiset yksikkökuutiot puun pituuden funktiona.

of the bark and of the increment during  $n$  years are calculated as differences. The three basic volumes are calculated by the formulas:

$$V/\text{ha} = G/\text{ha} \times \bar{V}_u \quad (9)$$

$$V/\text{ha}, \text{ e.b.} = G/\text{ha}, \text{ e.b.} \times \bar{V}_{u, \text{ e.b.}} \quad (10)$$

$$V/\text{ha}, -n, \text{ e.b.} = G/\text{ha}, -n, \text{ e.b.} \times \bar{V}_{u, -n} \quad (11)$$

The unit volume ( $V_u$ ) is the key characteristic in volume calculation. It possesses an important feature which is illustrated in Fig. 1. From a volume table with diameter and height as variables, the volumes of pine and spruce (Tapion taskukirja, 1959) are calculated into unit volumes (4) and expressed as the function of the tree height. The unit volume is very close to being directly proportional to the tree height when  $h \geq 3 \text{ m}$ . It can be calculated by the formula:

$$V_u = 2.00 \text{ m}^3/\text{ha.} + (h - 3 \text{ m.}) \times 0.45 \text{ m}^3/\text{ha.} \quad (12)$$

Consequently:

1.  $\bar{V}_u$  and  $\bar{h}$  can be estimated on the basis of a random or systematic sample from the trees tallied with the relascope. The estimates of  $\bar{V}_u$  and  $\bar{h}$  are arithmetic means of the sample-tree characteristics and their statistical precision is depend-

ent upon the number the sample trees ( $n$ ), the variation of  $V_u$  and  $h$ , and the proportion of  $n/N$ .

2.  $V/\text{ha}$ , e.b. is a product of  $V_u$ .

3.  $\bar{V}_{u,-n}$ , e.b. can be calculated from the equation:

$$\frac{\bar{V}_{u,-n, \text{e.b.}}}{\bar{h}_{-n}} = \frac{\bar{V}_u}{\bar{h}}$$

$$\bar{V}_{u,-n, \text{e.b.}} = \frac{\bar{h}_{-n} \times \bar{V}_u}{\bar{h}} \quad (13)$$

In respect to tree form, it is supposed that the bark and the diameter increment are proportionally the same on every part of the stem.

#### Symbols and units of measure — Merkit ja mittayksiköt

U unit tree — *yksikköpuu*

N number of unit trees — *yksikköpuiden lukumäärä*

#### Unit-tree characteristics — *Yksikköpuun tunnukset*

$d$  (cm.) diameter at breast height — *läpimitta rinnantasalla*

$g$  ( $\text{m}^2$ ) basal area — *pohjapinta-ala*

$h$  (m.) height — *pituus*

$f$  (cm.) form expressed as tapering class — *muoto kopenemisluokkana*

$v$  ( $\text{m}^3$ ) volume — *kuutio*

$\bar{h}$  (m.) arithmetic mean height — *aritmeettinen keskipituus*

$d.b.$  (cm.) double-bark thickness at breast height — *kaksinkertainen kuoren paksuus rinnantasalla*

$i_d$  (cm.) diameter increment — *läpimitan kasvu*

$i_h$  (m.) height increment — *pituuden kasvu*

$\bar{i}_h$  (m.) arithmetic mean of height increments — *pituuskasvujen aritmeettinen kesiarvo*

#### Unit characteristics — *Yksikkötunnukset*

$N_u$  unit number of trees per hectare — *yksikkörunkoluku hehtaarilla*

$G_u$  ( $\text{m}^2/\text{ha}$ ) unit basal area — *yksikköpohjapinta-ala*

$V_u$  ( $\text{m}^3/\text{ha}$ ) unit volume — *yksikkökuutio*

$\bar{G}_u$  ( $\text{m}^2/\text{ha}$ ) arithmetic mean of unit basal areas — *yksikköpohjapinta-alojen aritmeettinen kesiarvo*

$\bar{V}_u$  ( $\text{m}^3/\text{ha}$ ) arithmetic mean of unit volumes — *yksikkökuutioiden aritmeettinen kesiarvo*

#### Additional symbols — Lisämerkkijää

i.b. including bark — *kuorineen*

e.b. excluding bark — *kuoretta*

$n$  number of years or sampling units — *vuosien tai otannan yksikköjen lukumäärä*

$-n$   $n$  years ago — *n vuotta sitten*

Examples — *Esimerkkejä*:  $V_u$ , e.b. unit volume excl. bark — *yksikkökuutio kuoretta*;  $G_{u,-n}$ , e.b. unit basal area  $n$  years ago excl. bark — *yksikköpohjapinta-ala n vuotta sitten kuoretta*.

#### Characteristics of the stand to be estimated — Arvioitavan metsän tunnukset

$N/\text{ha}$  number of trees per hectare — *runkoluku hehtaarilla*

$G/\text{ha}$  ( $\text{m}^2/\text{ha}$ ) basal area per hectare — *pohjapinta-ala hehtaarilla*

$V/\text{ha}$  ( $\text{m}^3/\text{ha}$ ) volume per hectare — *kuutio hehtaarilla*

#### Calculation procedures

In Table 1 the unit number appears as a function of the diameter class and the number of unit trees (2), and in Table 2 the unit basal area excl. in  $1/100 \text{ m}^2$  now and  $n$  years ago is a function of the double-bark thickness and diameter increment during  $n$  years (6).

A relascope sample plot measured in a spruce stand is used in the following calculation example. Table 3 is the relascope stock table of the plot. Other data is collected in Table 4.

The number of the unit trees ( $N$ ) is 24 and the estimate of the number of trees per hectare ( $N/\text{ha}$ ) is 338. Diameter, height and form of each unit tree are measured. Thus the number of the volume sample trees is 24. The form is expressed as a tapering class which is the difference  $d_{1.3} \text{ m.} - d_{6} \text{ m.}$  (ILVES-SALO 1948). Bark and 5-year diameter and height increments are measured from 12 sample trees ( $n = 12$ ) selected systematically from the tallied unit trees.

Table 1. Unit numbers per hectare as a function of the unit-tree diameter.  
**Taulukko 1.** Yksikkörunkoluku hehtaarilla yksikköpuun läpimitan funktiona.

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d	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	
$N_u$	•	56..5	49..8	44..1	39..4	35..2	31..8	28..9	26..3	24..1	22..1	20..4	18..8	17..5	16..2	15..1	14..1	13..2	12..4	11..7	11..0	10..49..82..9..36..8..82..8..37..7..96..7..58..						
2 × »	•	113	100	88	79	70	64	58	53	48	44	41	38	35	32	30	28	26	25	23	22	21	20	19	18	17	16	15
3 × »	•	170	149	132	118	106	95	87	79	72	66	61	56	53	49	45	42	40	37	35	33	31	29	28	26	25	24	23
4 × »	•	226	199	176	158	141	127	116	105	96	88	82	75	70	65	60	56	53	50	47	44	42	39	37	35	33	32	30
5 × »	•	282	249	221	197	176	159	145	132	120	111	102	94	87	81	76	71	66	62	58	55	52	49	47	44	42	40	38
10 × »	•	565	498	441	394	352	318	289	263	241	221	204	188	175	162	151	141	132	124	117	110	104	98	93	88	84	80	76

Table 2. Unit basal area ( $G_u$ ) in  $1/100 \text{ m}^2$  excl. bark and  $n$  years ago excl. bark as a function of double-bark thickness (d.b.) and summation of double bark and diameter increment

Taulukko 2. Yksikköpohjapinta-ala ( $G_1$ ) /100 m<sup>2</sup>-ssä kuoretta ja vuotta sitten kuorettaa katsinkertaisen kuoren vahvuuden (a.b.) ja kaksinkertaisen kuoren vahvuuden sekä läpimittan kasvun summan (d.b. + i<sub>d</sub>) funktiona.

Table 3. Relascope stock table of a spruce stand.  
Taulukko 3. Kuusikkokoealan relaskooppipuusto.

d	17	23	25	27	29	31	35	33	37	39	41	$\Sigma$
N	1	2	1	2	3	2	4	3	2	3	1	24
$N_u$	44.1	24.1	20.4	17.5	15.1	13.2	11.7	10.4	9.30	8.37	7.58	
N/ha	44	48	20	35	45	26	47	31	19	25	8	338

The necessary mean characteristics are calculated in Table 4. Mean height of the unit trees ( $\bar{h}$ ) is 26.83 m., mean height increment ( $\bar{i}_h$ ) 0.34 m. and the estimate of the mean height 5 years ago ( $\bar{h}_{-5}$ ) 26.49 m. The unit volume is calculated by formula (4),  $V_u = N_u \times v$ , where  $v$  is the volume of each unit tree from the volume tables of ILVESSALO (1948). These tables have diameter, height and tapering class as variables. The summation of the unit volumes is 292.2 m<sup>3</sup>/ha. and the mean unit volume ( $\bar{V}_u$ ) is 12.18 m<sup>3</sup>/ha.

The unit basal areas excl. bark now ( $G_u$ , e.b.) and 5 years ago ( $G_{u,-5}$ , e.b.) as represented by each bark and increment sample tree are taken from Table 2. The estimate of the present mean unit basal area excl. bark ( $\bar{G}_u$ , e.b.) is 0.8825 m<sup>2</sup>/ha. and of the mean unit basal area 5 years ago excl. bark ( $\bar{G}_{u,-5}$ , e.b.) is 0.8133 m<sup>2</sup>/ha.

The mean unit volume 5 years ago ( $\bar{V}_{u,-5}$ ) is calculated by the formula (13):

$$\bar{V}_{u,-5} = \frac{\bar{h}_{-5} \times \bar{V}_u}{\bar{h}} = \frac{26.49 \times 12.18}{26.83} = 12.03 \text{ m}^3/\text{ha.}$$

Estimates of the basal area incl. bark (G/ha), excl. bark (G/ha, e.b.) and 5 years ago excl. bark (G/ha, -5, e.b.) for the sample plot stand are:

$$G/\text{ha} = N = 24.00 \text{ m}^2/\text{ha.}$$

$$G/\text{ha, e.b.} = N \times \bar{G}_u, \text{e.b.} = 24 \times 0.8825 = 21.18 \text{ »}$$

$$G/\text{ha, -5, e.b.} = N \times \bar{G}_{u,-5}, \text{e.b.} = 24 \times 0.8133 = 19.52 \text{ »}$$

Because all the unit trees are measured as volume sample trees, the summation of the unit volumes (5), 292.2 m<sup>3</sup>/ha., is the plot volume. If the number of the volume sample trees is less than the number of the unit trees, the estimate of the plot volume incl. bark can be calculated on the basis of the mean unit volume and by the formula (9). In this case the plot volumes incl. bark, excl. bark, and 5 years ago excl. bark calculated by the formulas (9), (10) and (11) are:

$$V/\text{ha} = G/\text{ha} \times \bar{V}_u = 24 \times 12.18 = 292.3 \text{ m}^3/\text{ha.}$$

$$V/\text{ha, e.b.} = G/\text{ha, e.b.} \times \bar{V}_u = 21.18 \times 12.18 = 258.0 \text{ »}$$

$$V/\text{ha, -5, e.b.} = G/\text{ha, -5, e.b.} \times \bar{V}_{u,-5} = 19.52 \times 12.03 = 234.8 \text{ »}$$

Table 4. Calculation of the mean characteristics from plot data.  
Taulukko 4. Keskikustannusten laskenta mittaustiedoista.

N = 24 Volume sample trees Kuutiohoepuita							Bark and increment sample trees Kuori- ja hasvukoepuita				
Plot data — Mittaustiedot							Calculation — Laskenta				
d	h	f	d.b.	$i_d$	$d.b. + i_d$	$i_h$	$N_u$	v	$V_u$	$G_u, \text{e.b.}$	$G_{u,-5, \text{e.b.}}$
34	30	5	1.9	1.4	3.3	.35	11.0	1.21	13.3	.89	.81
32	27	4	—	—	—	—	12.4	1.02	12.6	—	—
40	33	8	1.9	2.1	4.0	.30	7.96	1.52	12.1	.91	.81
28	26	3	—	—	—	—	16.2	.790	12.8	—	—
16	15	3	1.0	1.0	2.0	.30	49.8	.156	7.8	.88	.77
35	30	5	—	—	—	—	10.4	1.29	13.4	—	—
29	29	2	1.6	1.2	2.8	.45	15.1	1.00	15.1	.89	.81
23	24	4	—	—	—	—	24.1	.446	10.7	—	—
32	27	7	2.1	1.2	3.3	.30	12.4	.834	10.3	.87	.80
29	26	4	—	—	—	—	15.1	.797	12.0	—	—
23	25	2	1.6	.6	2.2	.35	24.1	.540	13.0	.87	.82
33	26	4	—	—	—	—	11.7	1.06	12.4	—	—
36	27	5	2.0	1.2	3.2	.45	9.82	1.24	12.2	.87	.83
31	28	4	—	—	—	—	13.2	.985	13.0	—	—
41	29	6	2.1	1.4	3.5	.35	7.58	1.65	12.5	.90	.84
25	21	5	—	—	—	—	20.4	.454	9.3	—	—
35	28	4	2.0	1.1	3.1	.40	10.4	1.28	13.3	.89	.83
39	32	6	—	—	—	—	8.37	1.62	13.6	—	—
27	23	2	1.7	.6	2.3	.25	17.5	.697	12.2	.88	.84
34	30	7	—	—	—	—	11.0	1.05	11.6	—	—
28	24	3	2.4	1.0	3.4	.30	16.2	.736	11.9	.83	.77
34	27	5	—	—	—	—	11.0	1.10	12.1	—	—
39	30	6	2.3	1.2	3.5	.25	8.37	1.53	12.8	.89	.83
36	27	5	—	—	—	—	9.82	1.24	12.2	—	—
$\Sigma$	644					4.05			292.2	10.59	9.76
$\bar{h} = 26.83$						$\bar{i}_h = .34$			$\bar{V}_u = 12.18$	$\bar{G}_u, \text{e.b.} = .8825$	$\bar{G}_{u,-5, \text{e.b.}} = .8133$
$\bar{h}_{-5} = 26.49$											

The volume of the bark is 34.3 m<sup>3</sup>/ha. (292.3—258.0) and it comprises 11.7 per cent of the volume incl. bark. Volume increment during 5 years excl. bark is 23.2 m<sup>3</sup>/ha. (258.0—234.8), mean annual increment 4.64 m<sup>3</sup>/ha. and increment is 1.80 per cent of the present volume excl. bark.

The mean unit volumes can also be calculated with the volume line equation (12). The mean heights needed are found from Table 4.:

$$\bar{V}_u = 2.00 + (\bar{h}-3) \times 0.45 = 2.00 + (26.83-3.00) \times 0.45 = 12.72 \text{ m}^3/\text{ha.}$$

$$\bar{V}_{u,-5} = 2.00 + (\bar{h}_{-5}-3) \times 0.45 = 2.00 + (26.49-3.00) \times 0.45 = 12.57 \text{ »}$$

The mean unit volumes multiplied by the corresponding basal areas give estimates of the plot volume. The bark and increment are differences as above:

volume incl. bark .....	305.3 m <sup>3</sup> /ha.
» excl. » .....	269.4 »
» 5 years ago excl. bark .....	245.4 »
bark .....	35.9 »
increment during 5 years.....	24.0 »
annual increment .....	4.80 »
bark per cent .....	11.8 »
increment per cent .....	1.78 »

Volume, bark and increment estimation based diameter, height and volume line equation is much faster than the estimation based diameter, height and form. The latter is obviously more accurate. In this case the volume estimate based the volume line equation is greater because the volume line equation presupposes better form than the trees of the plot stand possess.

#### Precision of the estimates

Because all unit trees are measured as volume sample trees, there is no statistical error in the plot volume.

The mean unit volume ( $\bar{V}_u$ ) based on all 24 unit trees is 12.18. The variance ( $s^2$ ) is 2.20 ( $\sum V_u^2 = 3608.18$  and  $(\sum V_u)^2 = 85380.84$ ). Thus the estimate of the mean unit volume based on 5, 10, 15 or 20 objectively selected sample trees has the standard error ( $s_{\bar{V}_u}$ ), which is obtained from the equation

$$(s_{\bar{V}_u})^2 = s^2 \times \frac{N-n}{N \times n}$$

as follows:

n	$\pm s_{\bar{V}_u}$	per cent of the mean
5	0.59	4.8
10	0.36	3.0
15	0.24	2.0
20	0.14	1.1
24	0	0

Double standard error being allowable, the precision of the plot volume attained with 5 sample trees is about 10 %, with 10 sample trees about 6 %, etc. The estimate based on the 12 bark and increment sample trees is 293.0 m<sup>3</sup>/ha., or 0.3 % too great.

The estimate of the increment is based on 12 sample trees. The increments calculated as differences of the unit volumes are:

$$\Delta V_u \quad 1.2 \quad 1.3 \quad 1.1 \quad 1.3 \quad 0.8 \quad 0.8 \quad 0.9 \quad 1.0 \quad 0.9 \quad 0.5 \quad 0.8 \quad 0.9$$

The standard error of the increment is 5 %, or about twice as large as the standard error of the volume based on 12 sample trees.

#### Relascope plots in forest inventory

Accurate determination of a sample plot with the relascope requires checking the boundary trees with a tape. In an average forest there are 10 to 20 unit trees on each plot if the opening of the relascope is 2 cm. and the distance from the eye 1 m. The number of the boundary trees to be checked is from 1 to 3.

The number of trees to be tallied on the relascope plots is very economical compared with the number of trees to be tallied on the ordinary plots. For the growing stock described in Fig. 2, a relascope plot specified above is as efficient as the ordinary circular plot of 5 ares. The number of trees to be tallied on 10 ordinary plots and on 10 relascope plots is as follows:

	Number of trees	Number of trees having diameter more than 26 cm.
on ordinary plots .....	594	20
on relascope plots .....	125	33

If the plot size of 1 are is used for the trees having a diameter of less than 10 cm. and the size of 5 ares for the trees with a diameter of more than 10 cm., the total number of trees to be tallied on the ordinary plots is 244.

In Figures 3—5 are seen the unit volumes based on the sample trees of pine, spruce and birch measured in four different inventories. (The inventories were carried out in Southern Finland by the Institute of Forest Mensuration and Management at the University of Helsinki.) The linear correlation between the unit volume and tree height makes it possible to work out simple methods for calculating the volume of the tallied trees. An estimate of the mean unit volume of the tallied growing stock is the arithmetic mean of the unit volumes based on objectively selected sample trees. If the volume is calculated with a volume line equation, an estimate for the mean height of the tallied trees is the arithmetic mean of the sample tree heights. Calculation and tabulation of the results can be easily carried out with punched cards.

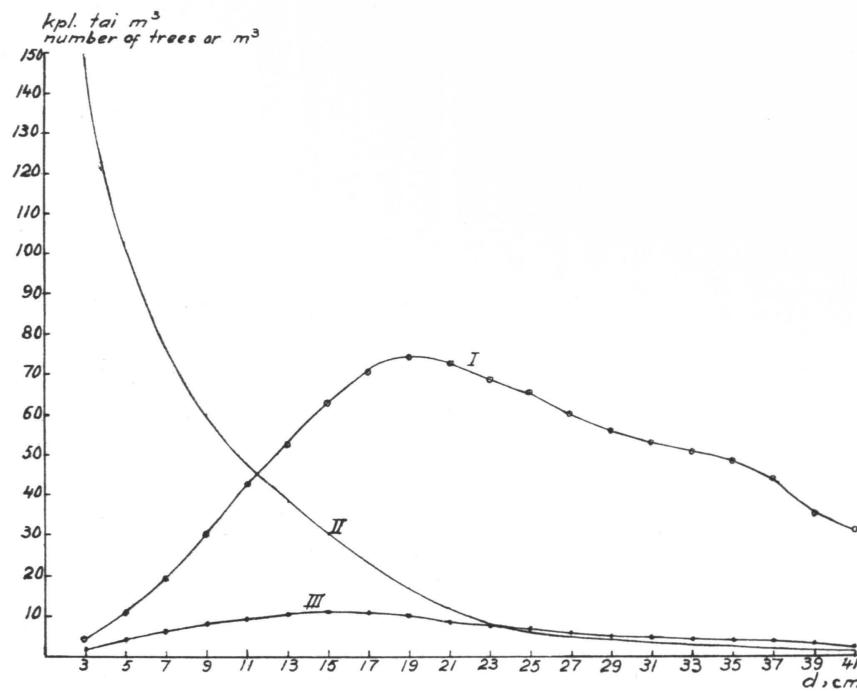
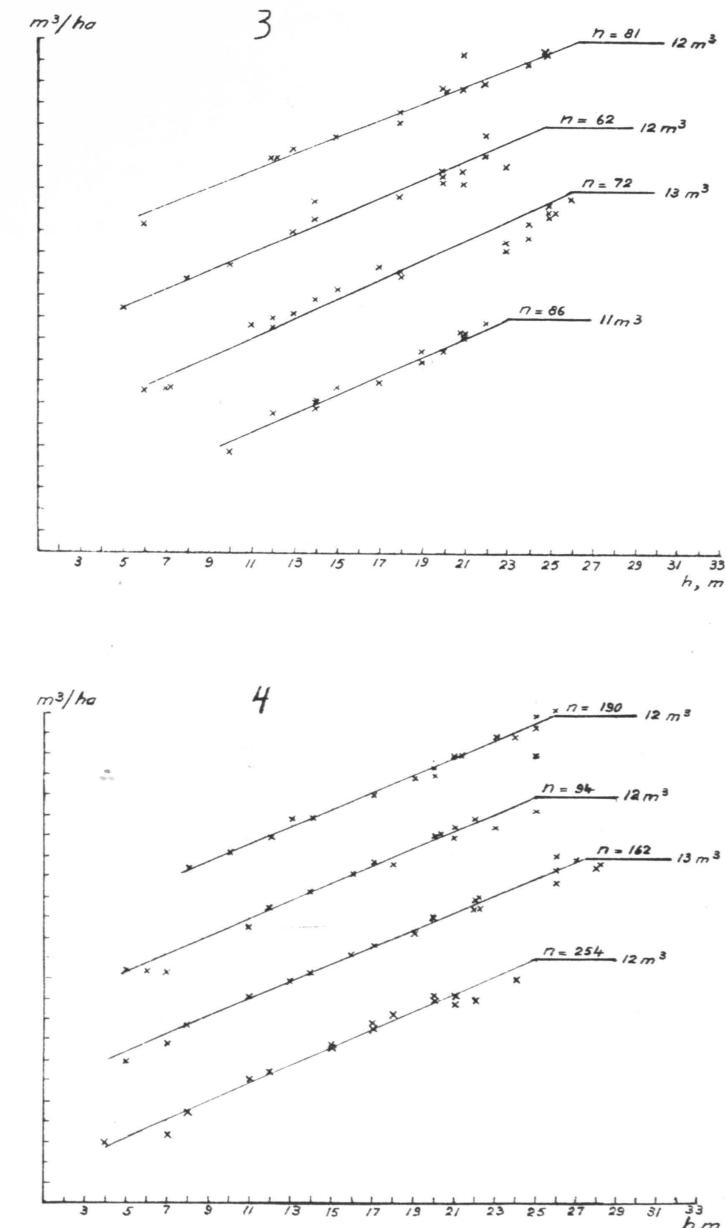


Fig. 2. Curve I: Distribution of the growing stock volume by diameter classes in a tract of forest, II: trees to be tallied on 10 sample plots of 5 ares each, III: trees to be tallied on 10 relascope sample plots.

Piirros 2. Käyrä I: Metsälalueen puuston kuutiomääärän jakaantuminen läpimittaluokkiin, II: 10:llä 5 aarin koealalla luettava puusto, III: 10:llä relaskooppikoealalla luettava puusto.

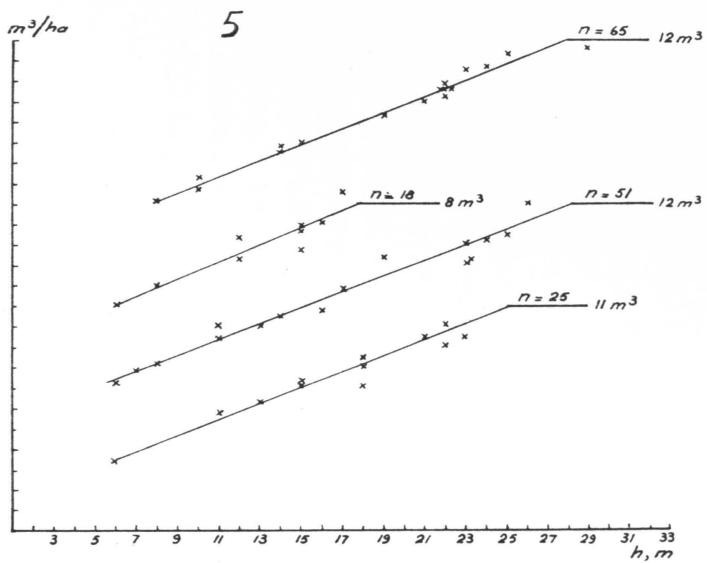
### Conclusions

Because all trees of equal diameter to be tallied on a sample plot, or on a number of sample plots, represent an equal share of the total basal area, the number of trees to be tallied is very economical from the standpoint of volume estimation. Objective selection of the sample trees can easily be done. The unit volume per hectare represented by each tallied tree, or by each sample tree, is directly proportional to the tree height. Thus the estimates of the stock characteristics can be calculated as arithmetic means from the sample tree characteristics. The calculation procedure which gives the ordinary stock table, volume, bark and increment is also easily carried out with punched cards.



Figures 3—5. Unit volumes ( $V_u$ ) as a function of the tree height based on pine, spruce and birch sample trees ( $n$  = number of trees) measured in four inventories. The points are mean unit volumes of height and diameter classes.

Piirrokset 3—5. Neljässä inventoinnissa mitattujen mänty-, kuusi- ja koivukoepuiden ( $n$  = kpl) mukaiset yksikkökuutiot ( $V_u$ ) puun pituuden funktioina. Pisteet ovat pituus- ja läpimittaluokkien keskiarvoja.

**SELOSTUS:****KUUTION JA KASVUN LASKENTA RELASKOOPILLA MÄÄRITETYLÄ KOEALALLA**

Jokainen relaskoopilla määritetyllä koealalla luettu yksikköpuu edustaa  $1\text{ m}^2$  pohjapinta-alaa sekä kokona mukaista runkolukua (2) ja kuutiota (4) hehtaarilla. Yksikköpuun edustamaa pohjapinta-alaa, runkolukua ja kuutiota kutsutaan yksikkötunnusksi (s. 8). Koealalla yksikkötunnusten summa on arvio metsikön vastaavasta tunnuksesta hehtaaria kohden: (1), (3) ja (5). Pohjapinta-ala kuoretta ja n vuotta sitten kuoretta lasketaan yksikköpuun vastaavien tunnusten perusteella samalla tavalla kuin kuorellinen nykypohjapinta-ala: (6) ja (7). Käytännön laskuja varten on edullista taulukoida sekä yksikkörunkoluku (Taulukko 1) ja yksikköpohjapinta-ala kuoretta ja n vuotta sitten kuoretta (Taulukko 2). Kun kuution ja kasvun laskenta tapahtuu koepuiden perusteella, joista on mitattu läpimitta, pituus ja muoto, tarvitaan keskimääräinen yksikkökuutio, keskimääräinen yksikköpohjapinta-ala kuoretta ja n vuotta sitten kuoretta sekä yksikköpuiden keskipituus n vuotta sitten (8). Jos laskenta perustuu vain läpimitan ja pituuden mittamiseen, saadaan keskimääräinen yksikkökuutio keskipitudoon ja asianomaiselle metsikölle sopivan kuutioimissuoran avulla.

Piirrosten 1 sekä 3–5 perusteella voidaan todeta, että yksikkökuutio muuttuu varsin tarkasti suorassa suhteessa puun pituuteen. Suhdetta ilmaisee kuutioimissuoran yhtälö (12), joka vastaa Tapion taskukirjan kuutioimistaulukon aineistoon mitattujen koepuiden muotoa. Keskimääräinen yksikkökuutio ja keskipituus voidaan siis laskea koepuiden vastaavien tunnusten aritmeettisina keskiarvoina, joiden tilastollinen tarkkuus on riippuvainen yksikköpuista otettujen koepuiden lukumäärästä, asianomaisen tunnuksen vaihtelusta ja otannan suhteesta n/N. Kuorettoman kuution laskeminen tapahtuu kuorellisen yksikkökuution perusteella ja keskimääräinen yksikkökuutio n vuotta sitten voidaan laskea kaavalla (13). Kuorettoman ja n vuotta sitten olleen kuorettoman kuution laskennassa joudutaan olettamaan, että kuoren paksuus ja läpimitan kasvu ovat suhteellisesti yhtä suuret rungon joka kohdalla.

Taulukossa 4 on esitetty kuusikossa mitatun koealan tiedot ja tarpeellisten keskitunnusten laskenta. Tarkin arvio saadaan käyttämällä kaavoja (4), (9), (11), (12) ja (13). Kuorellinen kuutio on  $292.2\text{ m}^3/\text{ha}$ , kuoreton  $258.0\text{ m}^3/\text{ha}$  ja kuutio 5 vuotta sitten kuoretta  $234.8\text{ m}^3/\text{ha}$ . Kuorisadannes on  $11.7$  (kuorelliseen nykypuuteen verrattuna), vuotuinen kasvu  $4.64\text{ m}^3/\text{ha}$  ja kasvusadannes  $1.80$  (kuorettomaan nykypuuteen verrattuna). Koska kaikki luetut puut on mitattu kuutiokoepuina, ei kuorellisessa kuutiolla ole tilastollista virhetä. 5 koepuulla olisi kuution arviossa noin 5 %:n, 10 koepuulla noin 3 %:n ja 20 koepuulla noin 1 %:n suuruinen keskivirhe.

Kuutioimissuora käytettäessä saadaan keskimääräinen yksikkökuutio kaavalla (12), joka edellyttää vastaavan keskipitudoon tuntemista. Tässä vaihtoehdossa ei puiden muodon vaietus kuutioon tule kuitenkaan huomioon otetuksi muutoin kuin keskimääräisenä.

Koska relaskooppikoealalla on eri kokoisia puita suorassa suhteessa niiden osuuteen arviotavan puiston pohjapinta-alasta, on puiden luku erittäin taloudellista kuution arvioimista silmällä pitäen (Piirros 2). Koska yksikkökuutio muuttuu suorassa suhteessa puun pituuteen, voidaan esitetyjä periaatteita käyttää kuutioitaessa myös useammalla kuin yhdellä relaskooppikoealalla luettuja puita. Tarpeelliset tunnukset voidaan laskea objektiivisesti otetuista koepuista aritmeettisina keskiarvoina, mikä jouduttaa laskelmien suorittamista ja yksinkertaistaa tilastokoneiden käytämistä.

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