Optimising the Management of a Heterogeneous Stand

Timo Pukkala and Jari Miina

Pukkala, T. & Miina, J. 2005. Optimising the management of a heterogeneous stand. Silva Fennica 39(4): 525–538.

The study presents a method for taking the heterogeneity of the stand into account in the optimisation of stand management. Heterogeneity refers to within-stand variation in stand density and/or other characteristics. A set of plots, corresponding to different sub-areas of the stand, represents the stand in calculations. Cuttings and other treatments of the plots are done simultaneously. The method was used to analyse how the optimal management depends on the heterogeneity of a Norway spruce (*Picea abies* (L.) Karst.) and Scots pine (*Pinus sylvestris* L.) stand. The results supported the hypothesis that the heterogeneity of a stand decreases its optimal prior-thinning density. Also the remaining stand basal areas were lower in heterogeneous stands, especially in spruce. The effect of stand heterogeneity prior to the first commercial thinning still affected the timing of the second thinning, which had to be conducted earlier and at lower prior-thinning basal areas in heterogeneous stands. This happened despite the fact that the first thinning greatly decreased the within-stand variation in stand basal area. In addition, heterogeneity decreased the soil expectation value, net income and timber harvests.

Keywords Hooke and Jeeves algorithm, thinning schedules, within-stand variation
Authors' addresses *Pukkala*, University of Joensuu, Faculty of Forestry, P.O. Box 111, FI-80101 Joensuu, Finland; *Miina*, Finnish Forest Research Institute, Joensuu Research Unit, P.O. Box 68, FI-80101 Joensuu, Finland **E-mail** timo.pukkala@joensuu.fi
Received 3 February 2005 **Revised** 10 May 2005 **Accepted** 19 October 2005
Available at http://www.metla.fi/silvafennica/full/sf39/sf394525.pdf

1 Introduction

A frequently used method to find the optimal management for a tree stand is the combined use of a stand simulator and an optimisation algorithm (see e.g. Hyytiäinen 2004 for references). The simulation program calculates the value of the objective variable with a given set of management parameters such as time points and intensities of cuttings. The optimisation algorithm alters the management parameters in a systematic way aiming at such a combination which maximizes the value of the objective variable. Management parameters that are optimised are called decision variables. The most common objective variable is the soil expectation value.

Optimisation of stand management is based on a single set of stand characteristics, a single plot, or a single set of sample trees (e.g. Monserud 1989). The optimisation results are often interpreted so that they tell the optimal way to manage stand. However, the interpretation of results is not so straightforward since stands vary in homogeneity and spatial distribution of trees. Kilkki et al. (1985) concluded that thinning instructions based on dominant height and stand basal area should be used differently in homogenous and heterogeneous stands since heterogeneous stands may have sub-areas that should be thinned even if the mean basal area of the stand is below the "thinning limit" of the instruction. This means that heterogeneous stands should be thinned at lower stand basal areas than homogeneous stands.

Pukkala (1990) calculated that when several systematically placed sample plots represent the stand, instead of only one average plot, and every plot is thinned according to silvicultural instructions (Hyvän metsänhoidon… 2001), the remaining mean stand basal area tends to be smaller in heterogeneous stands. Therefore, a heterogeneous stand will have a greater removal than a homogenous stand of the same mean basal area.

Some results calculated with distance dependent growth models (e.g. Pukkala 1989, Pukkala and Kolström 1991, Pretzsch 1995, Shao and Shugart 1997) or from several sample plots per stand (Pukkala 1990) suggest that the growth of a heterogeneous stand may be lower than the growth of a homogeneous stand with the same diameter distribution and other non-spatial stand characteristics. Accordingly, also the growth predictions of optimisation calculations may be biased for exceptionally homogeneous or heterogeneous stands if non-spatial models are used with one plot per stand.

This study presents a method that takes the heterogeneity of the stand into account in the optimisation of stand management when non-spatial models are used. In this study, heterogeneity refers to within-stand variation in stand density and/or other characteristics. The idea of the method is that a set of plots, corresponding to different sub-areas of the stand, represents the stand instead of a single plot. Cuttings and other treatments of all plots are done simultaneously. The method is used to analyse how the optimal management depends on the heterogeneity of a Norway spruce (*Picea abies* (L.) Karst.) and Scots pine (*Pinus sylvestris* L.) stand. It is hypothesized that both the prior-thinning and post-thinning basal areas are lower in heterogeneous stands.

2 Methods

2.1 Generation of Stand Data

The simulator that was employed in calculations used individual tree models. The input data of the simulator consisted of a list of representative trees (or sample trees), each tree being described with the following variables: species, age, diameter, height, and number of trees per hectare. Contrary to earlier studies, many sets of sample trees were generated for the stand, each set representing a different sub-area of the stand. Simulation for a given combination of decision variables was done with every set, i.e. as many times as there were sample tree sets. The mean value of the objective variable was returned to the optimisation algorithm.

The steps of the process of generating input data for the simulator were as follows:

- Specify the average values of the following stand variables: number of trees per hectare (N), stand basal area (G), basal-area-weighted mean diameter (D) and stand age (T)
- Generate sets of plot-level variables N, G, D and T by adding correlated multi-normal stochastic

variation to the mean values
- For each set:
  - Predict the diameter distribution of stand basal area
  - Draw 10 sample trees from the predicted distribution
  - Calibrate the frequencies of sample trees using goal programming
  - Predict the mean height (H)
  - Predict the height curve
  - Calculate the heights of the trees
  - Use the stand age of the set as the age of each sample tree of the set

The Cholesky decomposition (SPSS Inc. 2004) fitted to empirical data was used to generate multi-normally distributed variation in stand characteristics. The Cholesky decomposition was calculated for the following stand variables: G, ln$N$, D and T. The logarithmic transformation was taken of the number of stems (N) to have a normal distribution and linear relationships among stand variables. The Cholesky decomposition was based on the covariance matrix of the relative deviations of plot-level variables from the stand mean, i.e. the stand mean was subtracted from the plot-level variables and the results were divided by the stand mean. The relative plot-level variables and their covariance matrix were calculated using data from 318 fixed-radius plots (radius 4–20 m depending on the stage of stand development) systematically placed in 41 forest stands in eastern Finland (Anttila 2002), and 423 relascope plots (basal area factor 2 $m^2 ha^{-1}$) systematically placed in 25 stands in southern Finland (Sundström 2001).

To obtain stochastic and correlated values for G, ln$N$, D and T, four normally distributed random numbers with mean equal to zero and standard deviation equal to one were generated. The random numbers were multiplied with a variation multiplier to obtain stands of lower (multiplier < 1) or higher (multiplier > 1) variation than in the modelling data (Sundström 2001, Anttila 2002). The vector of scaled random numbers (s_1–s_4) was then multiplied with the Cholesky decomposition to get correlated relative deviations (r) for G, ln$N$, D and T:

$$
\begin{aligned}
r_G &= 0.33015\, s_1 \\
r_{\ln N} &= 0.03265\, s_1 + 0.06437\, s_2 \\
r_D &= 0.04749\, s_1 - 0.13901\, s_2 + 0.12915\, s_3 \\
r_T &= 0.03704\, s_1 - 0.08902\, s_2 + 0.07368\, s_3 \\
&\quad + 0.13844\, s_4
\end{aligned}
$$

The stochastic relative deviations were used to calculate a correlated set of stochastic stand variables:

$$
\begin{aligned}
G &= G_{mean} + r_G \times G_{mean} \\
\ln N &= \ln N_{mean} + r_{\ln N} \times \ln N_{mean} \\
D &= D_{mean} + r_D \times D_{mean} \\
T &= T_{mean} + r_T \times T_{mean}
\end{aligned}
$$

These values were used to generate a set of representative trees (sample trees) for the sub-area of the stand. The whole process of producing stochastic stand variables and generating a set of sample trees was repeated 100 times per variation multiplier, which means that every stand analysed in this study was represented by 100 sets of sample trees.

The generation of sample trees began with the prediction of the diameter distribution of stand basal area. The beta function was used as the theoretical distribution. The parameters of the beta distribution were calculated as explained in Loetsch et al. (1973). The variance of diameter, required in the calculation, was obtained from the model of Päivinen (1980). The minimum and maximum of the distribution were assumed to be 0.4D and 1.6D, respectively. The relative frequencies of 10 diameters were calculated from the predicted distribution (the range was divided into 10 intervals of equal width, and the frequency was calculated for the mid-point of every interval). The stand basal areas represented by the 10 sampled trees were scaled so that their sum was equal to stand basal area.

The frequencies obtained in this way were calibrated by solving the following optimisation problem with the Simplex method (see Deville and Särndal 1992, Kangas and Maltamo 2000):

$$\min \sum_{i=1}^{I} \left(s_i^- + s_s^+ \right) + 50N^- + 50N^+ + 50D^- + 50D^+$$

subject to

$$\sum_{i=1}^{I} g_i w_i = G$$

$$\sum_{i=1}^{I} w_i + N^- - N^+ = N$$

$$\sum_{i=1}^{I} d_i^2 (d_i - D) w_i + D^- - D^+ = 0$$

$$w_i + s_i^- - s_i^+ = f_i, \quad i = 1,...,I$$

where s_i^+ and s_i^- measure how much the calibrated frequency of diameter class i (w_i) exceeds (s_i^+) or falls short of (s_i^-) the non-calibrated frequency (f_i); N^+, N^-, D^+ and D^- are the corresponding goal variables for the total number of trees per hectare and mean diameter; I is the number of diameter classes; and g_i, w_i, f_i and d_i are, respectively, the tree basal area (m^2), calibrated frequency, non-calibrated frequency and mid-point diameter (cm) of diameter class i.

The purpose of calibration was to adjust the frequencies of diameter classes so that the total number of trees per hectare (N), stand basal area (G), and mean diameter (D), when calculated from the sample trees, agreed with the input values of the corresponding characteristics (Table 1). To be sure that the problem was always solvable, goal variables for N and D (N^+, N^-, D^+ and D^-) were added to the problem formulation. They were often zero because their weights in the objective function were 50 times greater than the weights of deviations of class frequencies. Calibration decreases the significance of the distribution function that is used to derive the first guesses of class frequencies. Because calibration does not generate new diameter classes, but often deletes diameter classes from one or both ends (Table 1), it is important that the pre-calibration distribution is wide enough.

The following models (Hossfeld formula) were used to estimate the basal-area-weighted mean height (H, m) of each sub-area of the stand:

Pine

$$H = (35.4 + 0.32G) / (1 + 29.3/D + 52.1/D^2) \qquad (1)$$

Table 1. Examples of the effect of calibration on the frequencies of diameter classes for four target combinations of stand basal area (G), number of trees per hectare (N) and basal-area-weighted mean diameter (D). Symbols d_i, f_i and w_i are, respectively, mid-point diameter (cm), non-calibrated frequency and calibrated frequency of diameter class i. The calibrated distribution (w_i) has G, N and D equal to the target values.

	$G = 26.47$ m^2 ha^{-1} $N = 2556$ ha^{-1} $D = 12.14$ cm									
d_i	5.6	7.0	8.5	10.0	11.4	12.9	14.3	15.8	17.2	18.7
f_i	258	492	521	472	395	311	228	151	82	23
w_i	0	105	521	472	480	598	228	151	0	0
	$G = 21.28$ m^2 ha^{-1} $N = 1840$ ha^{-1} $D = 12.42$ cm									
d_i	5.7	7.2	8.7	10.2	11.7	13.2	14.7	16.2	17.6	19.1
f_i	238	398	401	354	293	230	171	116	66	21
w_i	0	0	135	354	496	685	171	0	0	0
	$G = 22.00$ m^2 ha^{-1} $N = 2112$ ha^{-1} $D = 12.31$ cm									
d_i	5.7	7.1	8.6	10.1	11.6	13.1	14.5	16.0	17.5	19.0
f_i	243	416	422	374	310	244	181	122	69	22
w_i	0	188	422	374	310	504	181	122	11	0
	$G = 20.02$ m^2 ha^{-1} $N = 1662$ ha^{-1} $D = 14.91$ cm									
d_i	6.9	8.6	10.4	12.2	14.0	15.8	17.6	19.4	21.2	23.0
f_i	173	268	262	228	187	147	110	76	45	15
w_i	400	268	262	228	187	47	110	76	45	41

Spruce

$$H = (31.4 + 0.19G)/(1 + 13.4/D + 162.5/D^2) \quad (2)$$

where G is stand basal area ($m^2 ha^{-1}$) and D is mean diameter (cm) of the sub-area. The models are based on the compartment inventory data of the Sola region in North Karelia, Finland. The data are available in Anttila et al. (2001).

The mean diameter and mean height were used to predict the Näslund's height curve for the sub-area (Siipilehto 1999):

$$h = \frac{d^i}{(b_0 + b_1 d)^i} + 1.3 \quad (3)$$

where h is tree height (m), d is diameter (cm), $i = 2$ for pine and $i = 3$ for spruce. Parameter prediction models for the height curve (for parameter b_1 in Eq. 3; parameter b_0 was calculated from the mean diameter, mean height, and b_1) were obtained from Siipilehto (1999). The height curve was calibrated so that the mean height, as calculated from the sample trees, was equal to the mean height predicted with equation 1 or 2. The last step in generating the sample trees for a sub-area was to use the stand age of the sub-area as the age of each sample tree.

2.2 Simulation of Stand Development

The growth and survival of sample trees were simulated with the models of Hynynen et al. (2002). The site index model was used once per stand, and the following models for mineral soil once per 5-year time step: dominant height models, height growth models, self-thinning models, mortality models, crown ratio models, and basal area growth models. The simulation was done as explained in Hynynen et al. (2002). The volumes of timber assortments were calculated with the taper models of Laasasenaho (1982).

2.3 Decision Variables

The purpose of optimisation was to find such a cutting schedule for the stand that the soil expectation value is maximised. It was assumed that all sub-areas of a stand are treated simultaneously. Because of this, the timing of cuttings was based on the cutting year (for the first thinning) or number of years since previous cuttings (for the other cuttings) instead of stand basal area used in several earlier studies (e.g. Valsta 1992a,b, Pukkala et al. 1998).

Two options were considered when deciding how to specify the intensity of a thinning treatment. The first option was to specify the remaining stand basal area, and use the same value everywhere in the stand. With this option the harvest percentage was different in different sub-areas but the same relative thinning was used in all diameter classes of the sub-area.

The other option was to optimise a set of removal percentages and to use these percentages everywhere in the stand. This results in varying remaining basal areas: sparse sub-areas would have lower remaining basal areas than dense sub-areas. The removal percentage was specified for three diameters: minimum, mid-point and maximum of the diameter distribution (Valsta 1992a). Linear interpolation was used to get the removal percentage for the other diameters. Unequal removal percentages for different tree sizes allow one to optimise the type of thinning, in addition to thinning intensity.

In this study, the first option was used in the first thinning and the second option in the second thinnings. Using the second option also in the first thinning may give higher soil expectation values as it is more flexible and makes thinning from above possible already in the first commercial cutting. However, the first option (optimising remaining basal area with the same harvest percentage in every diameter class) was considered more reasonable in the first thinning because the opening of extraction roads and removing low-quality trees necessitates cutting all tree sizes. In addition, the same remaining basal area everywhere in the stand means that dense sub-areas are thinned more than sparse places, with a consequence that the thinning will decrease the heterogeneity of the stand, which was assumed to have a positive effect on stand productivity.

Only management schedules with two thinnings were optimised in this study. Therefore, the set of optimised decision variables was as follows:

1st thinning:

- Thinning year expressed as the number of years

since the average regeneration year of the stand
- Remaining basal area

2nd thinning
- Number of years since the first thinning
- Harvest percentage for the minimum diameter
- Harvest percentage for the mid-point of the diameter range
- Harvest percentage for the maximum diameter

Final felling
- Number of years since the second thinning

2.4 Economic Parameters

The stand regeneration cost was assumed to be 600 €ha^{-1}, and a tending cost of 200 €ha^{-1} was assumed 15 years after regeneration. The roadside prices of timber assortments were taken as follows (€m^{-3}): pine log, 48; pine pulpwood, 25; spruce log, 44; and spruce pulpwood, 31. The entry cost of cuttings was 10 €ha^{-1}. The variable costs of cuttings were calculated using the models of Rummukainen et al. (1995).

2.5 Initial Stands

Pine stands growing on rather poor site (VT) and a spruce stands on medium site (MT) were used in calculations. The temperature sum, which affects site index, was assumed to be 1100 d.d. The proportion of lakes within 20 km was 0.2, the proportion of see was zero, and the elevation of the site was 50 m a.s.l. The mean stand characteristics were the same in all pine stands and all spruce stands (Table 2), but the stand heterogeneity was varied by using the following variation multipliers: 0, 0.5, 1, 1.25, and 1.5. The characteristics of initial stands were chosen so that the first commercial thinning should be conducted soon but not immediately when soil expectation value with 2% discounting rate was maximised. This enabled us to investigate whether the optimal timing and intensity of the approaching first thinning was related to the heterogeneity of the stand.

Table 2. Mean values of stand characteristics in the analysed stands. Mean diameter refers to basal-area-weighted mean.

Variable	Pine stand	Spruce stand
Stand basal area, m^{2}ha^{-1}	20.0	27.0
Number of trees per hectare	1800.0	3000.0
Mean diameter, cm	13.0	11.5
Stand age, years	40.0	40.0

2.6 Optimisation

The method of Hooke and Jeeves (1961) was used to find the optimal combination of decision variables. The direct search of Hooke and Jeeves was repeated twice for every problem and the better solution is reported. The first search began from subjectively set initial values of decision variables and the second from the best of 1000 random combinations of decision variables. The random values were uniformly distributed within subjectively specified ranges. The initial step of changing the decision variables was 0.2 times the range of variation. The step size was gradually decreased during the search, and the search was terminated when the step size of every decision variable was smaller than 0.02 times the initial step.

The primary objective variable was soil expectation value with 2% discounting rate. However, 1% and 3% rates were also used to study the sensitivity of results to discounting rate. Another part of the sensitivity analysis examined whether the conclusions would change if mean annual harvested volume or mean annual net income was used as the objective variable (maximum sustained yield and maximum forest rent goals, respectively).

3 Results

3.1 Maximising Soil Expectation Value with 2% Discounting Rate

In the pine stand the optimal time of the first thinning was the earlier the higher was the within-stand variation in stand characteristics (Fig. 1A). The effect of initial heterogeneity still affected the timing of the second thinning (Fig. 1A) although

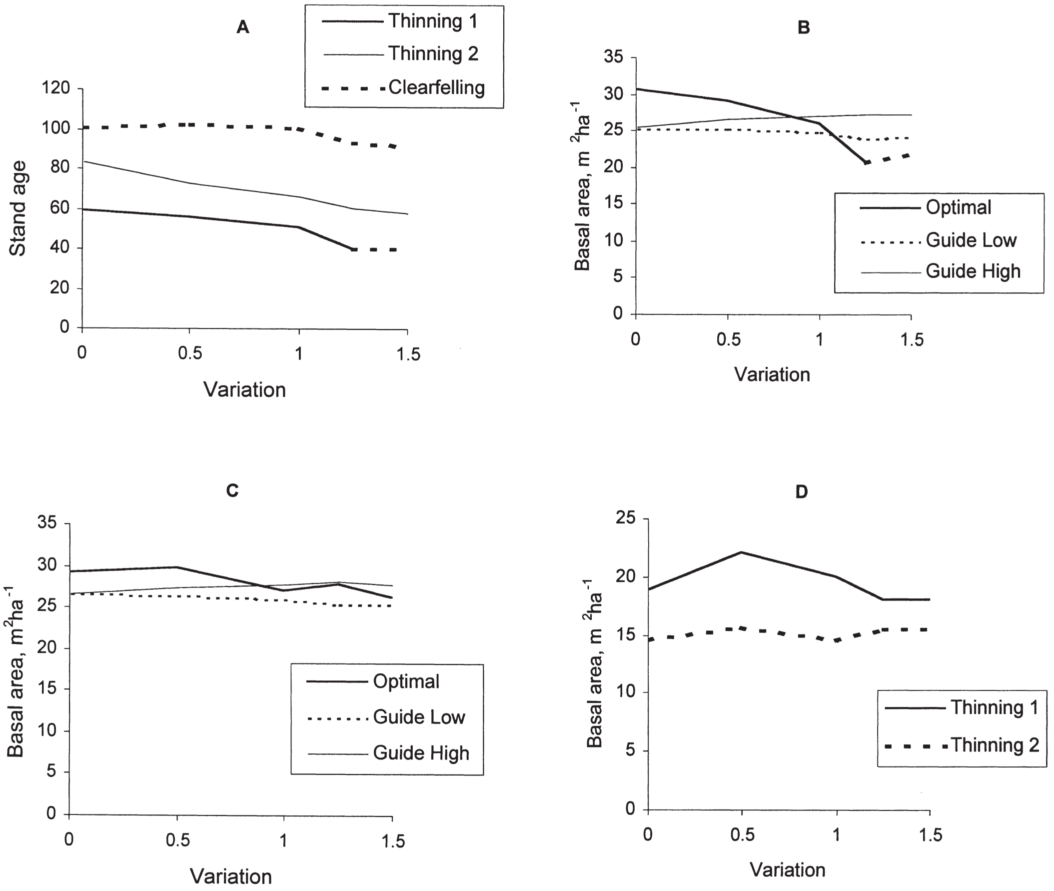

Fig. 1. Dependence of the optimal cutting age (A), stand basal area at the first thinning (B), stand basal area at the second thinning (C) and remaining stand basal area (D) on the within-stand variation in a pine stand. With variation multipliers 1.25 and 1.5 the optimal time of the first thinning was before the age of the initial stand (dashed part of the thick solid line in sub-figures A and B).

the first thinning largely eliminated the within-stand variation in stand basal area (Fig. 2). However, the interval between the first and second thinning did not depend much on stand heterogeneity. The optimal rotation length was nearly the same for all variation multipliers, but very high heterogeneity tended to shorten the rotation slightly.

The stand basal area at first thinning decreased with increasing heterogeneity (Fig. 1B). With variation multipliers 1.25 and 1.5 the first thinning should be concluded immediately (the optimal time had passed already before the age of the initial stand), which means that the decrease in thinning basal area as a function of heterogeneity would in fact be steeper than Fig. 1B suggests.

In a homogenous stand the optimal prior-thinning basal area was higher than in the thinning instruction for private forestry (Hyvän metsänhoidon… 2001) but in a heterogeneous stand the opposite was true. The basal area instructions of Fig. 1B and 1C (Guide Low and Guide High) were obtained by calculating the stand dominant height at the onset of thinning, and then taking the thinning limit corresponding to this dominant height from the instructions. “Guide Low” used

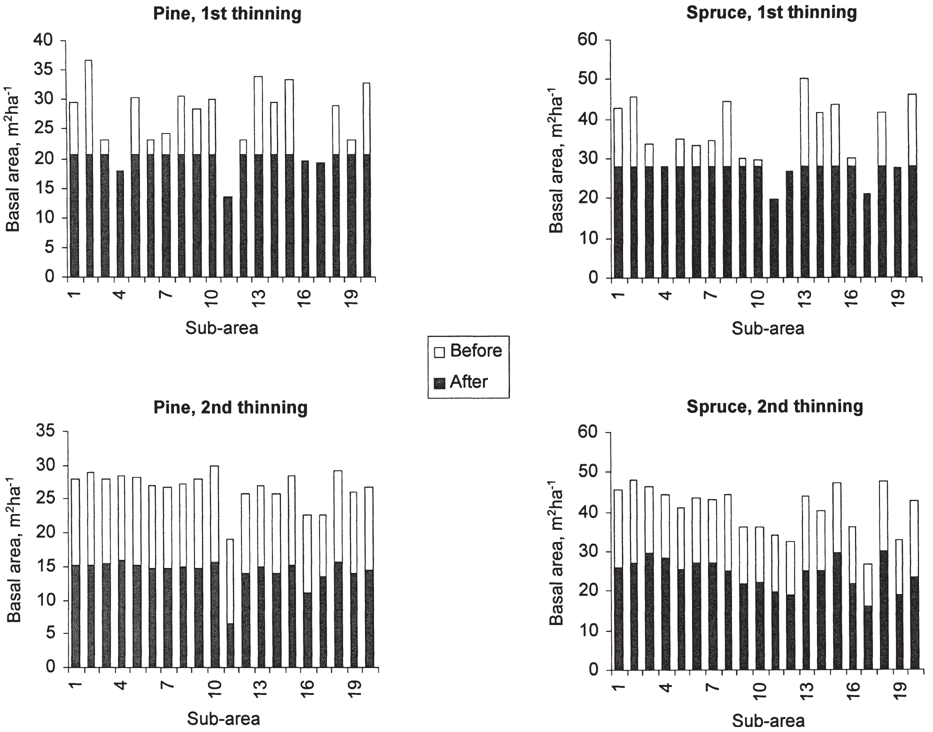

Fig. 2. Prior- and post-thinning basal areas of 20 plots (out of 100) that were used in the optimisation of the management of a heterogeneous stand (variation multiplier 1). Soil expectation value with 2% discounting rate was maximised.

the mean dominant height of the 100 plots that represented the stand while “Guide High” was based on the maximum dominant height among the 100 plots. The dominant height that a forester uses when applying the thinning instructions is certainly between these two values.

The basal area at the second thinning was no longer distinctly different for homogeneous and heterogeneous stands (Fig. 1C), most probably because the first thinning greatly decreased within-stand variation in heterogeneous stands (Fig. 2). The prior-thinning basal area was close to the thinning instructions. The remaining basal area did not change systematically as a function of stand heterogeneity (Fig. 1D).

The mean annual harvest of the optimal management schedule did not decrease with increasing heterogeneity (Fig. 3); it was always about 3.9 $m^3 ha^{-1}$. However, the mean annual net income (forest rent) and soil expectation value decreased with increasing heterogeneity.

Also in spruce the first thinning was earlier in heterogeneous stands (Fig. 4A) but later on the differences in the timing of cutting treatments decreased so that the year of final felling no longer correlated with the heterogeneity of the initial stand. The basal area at both thinnings decreased steeply with increasing heterogeneity of the initial stand (Fig. 4B and 4C). Contrary to pine, the effect of initial heterogeneity was still clear in the second thinning (Fig. 4C) although the first thinning largely equalised the basal areas of the 100 plots that represented the stand (Fig. 2). The optimal prior-thinning basal areas of homogeneous stands (with 2% discounting rate) were much higher than recommended in the instructions. The first thinning of a very heterogeneous stand should be done according to the silvicultural instructions (Hyvän metsänhoidon… 2001), but the second thinning at a higher-than-instructed basal area. In

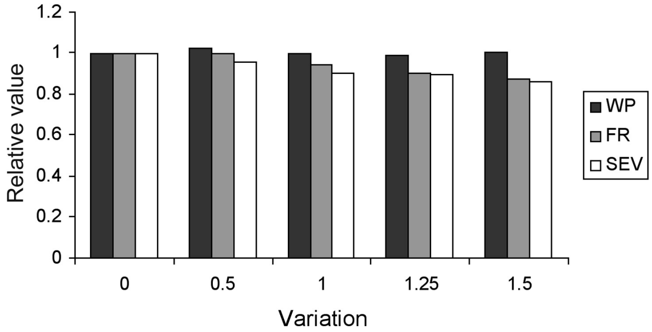

Fig. 3. Relative mean annual harvest (WP), mean annual net income (FR) and soil expectation value with 2% discounting rate (SEV) in pine stands of varying within-stand variation. The values are expressed as the proportion of the value for variation multiplier 0.

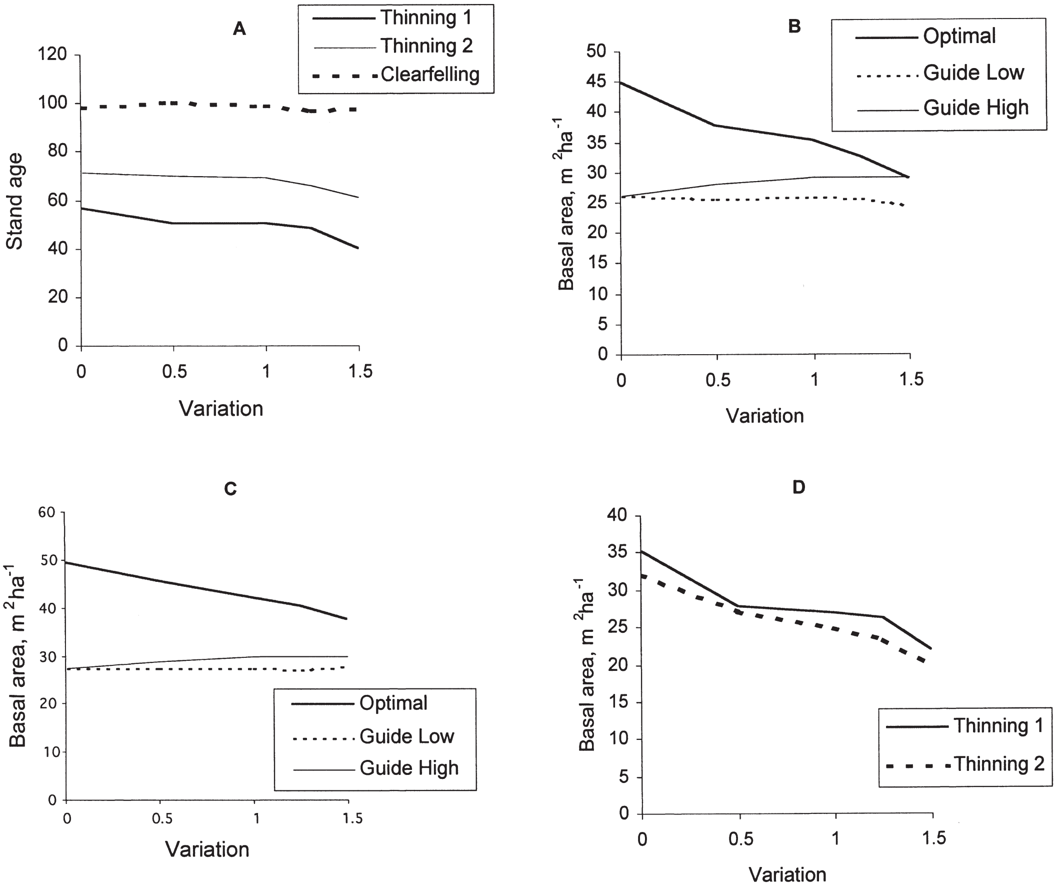

Fig. 4. Dependence of the optimal cutting age (A), stand basal area at the first thinning (B), stand basal area at the second thinning (C) and remaining stand basal area (D) on the within-stand variation in a spruce stand.

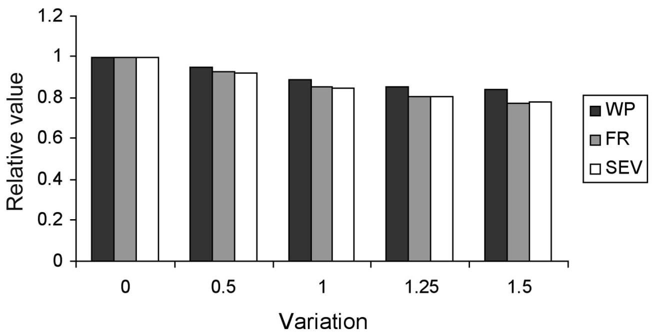

Fig. 5. Relative mean annual harvest (WP), mean annual net income (FR) and soil expectation value with 2% discounting rate (SEV) in spruce stands of varying within-stand variation. The values are expressed as the proportion of the value for variation multiplier 0.

Table 3. The loss in soil expectation value (with 2% discounting rate) if the optimal management schedule of a homogeneous stand is applied in heterogeneous stands. The loss is expressed as percent of the soil expectation value of the optimal management schedule of the heterogeneous stand.

Species	Variation multiplier			
	0.5	1.0	1.25	1.5
Pine	5.7	7.0	7.7	10.8
Spruce	0.5	2.6	2.8	2.6

spruce the optimal remaining basal area decreased as a function of within-stand variation (Fig. 4D), which was not the case in pine.

The mean annual harvest of the spruce stand decreased from 7.0 $m^3 ha^{-1}$ to 5.9 $m^3 ha^{-1}$ when the variation multiplier increased from zero to 1.5 (Fig. 5). The mean annual net income and the soil expectation value of the optimal management schedule decreased relatively faster than wood production with increasing heterogeneity.

If the optimal management schedule of the homogeneous pine stand was applied in non-homogeneous stands, the loss in soil expectation value was 5.7 to 10.8% (Table 3). The loss increased with increasing heterogeneity. In spruce the losses were much smaller, suggesting that it did not matter so much in spruce whether the prior- and post- thinning basal areas deviated from their optimal values.

3.2 Effect of Discounting Rate

The effect of stand heterogeneity on optimal stand management was similar as explained above also with 1% and 3% discounting rates: a heterogeneous stand was thinned at a lower stand basal area than a homogeneous stand (Fig. 6). The same trend could still be seen in the second thinning, especially in spruce. The optimal management schedule of a homogeneous pine stand had only one thinning when the discounting rate was 3%. In spruce, the remaining basal area was lower in heterogeneous stands than in homogeneous stands with all discounting rates. However, this trend cannot be seen in the results for pine. As expected, the optimal rotation became shorter with increasing discounting rate.

3.3 Effect of Objective Variable

The conclusions about the effect of heterogeneity did not change much when wood production (sustained yield) or net income (forest rent) was maximised (Figs. 7 and 8). However, there were some differences from earlier results. When wood

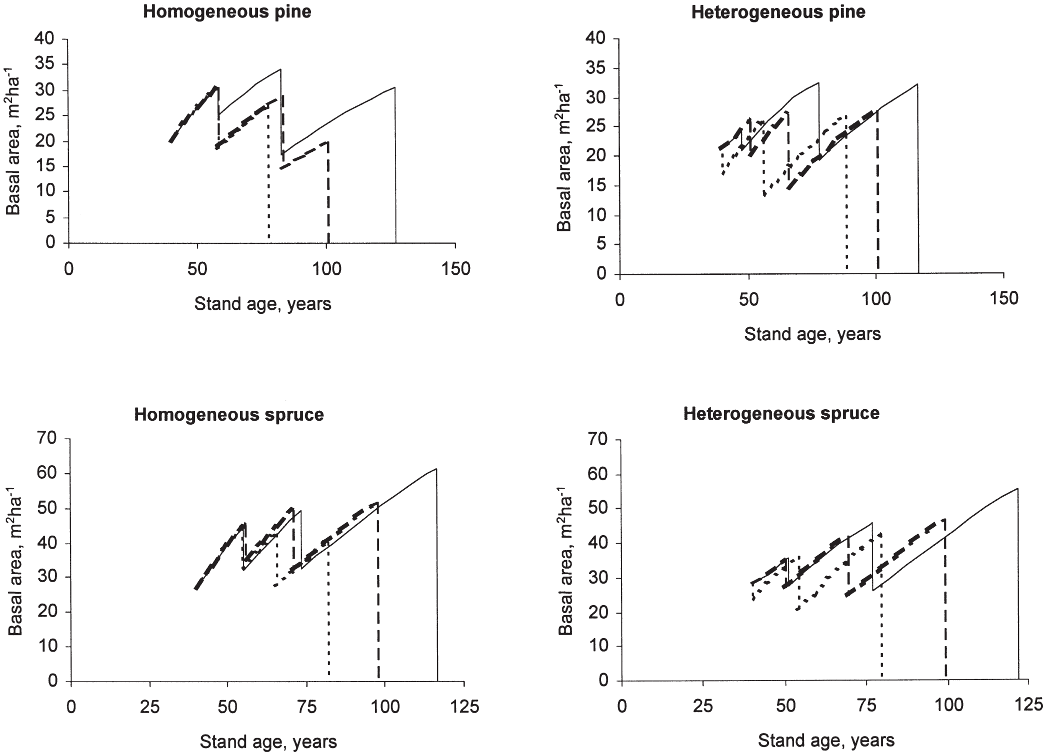

Fig. 6. Development of stand basal area in the optimal management schedule in a homogeneous (variation multiplier 0) and heterogeneous (variation multiplier 1) stand with discounting rates of 1% (solid line), 2% (dashed line) and 3% (dotted line).

production was maximised in pine, there was no removal in the first thinning of a homogeneous stand, i.e. the first thinning was actually the one that used harvest percentages as decision variables. In the other cases the remaining basal area was lower in a heterogeneous stand than in a homogeneous one (Fig. 7), which is different from the results for the SEV (soil expectation value) goal. The effect of initial heterogeneity of pine stand extended to the second thinning, in which both the prior- and post-thinning basal areas were lower for the heterogeneous stand.

In spruce, the remaining basal area of the first thinning was the same for the homogeneous and heterogeneous stand when net income or wood production was maximised (Fig. 8), which was not the case when soil expectation value was maximised. Otherwise the results on the effect of heterogeneity were similar as with the SEV goal, although the effect of heterogeneity appears to be somewhat smaller with the wood production or net income goals.

3.4 Effect of Decision Variables

The last part of the sensitivity analysis inspected the effect of the choice of decision variables on soil expectation value. As explained above, the remaining stand basal area, equal in all plots that represented the stand was used in the first thinning (with an equal harvest percentage in all diameter classes of a plot). In the second thinning, harvest percentages for the minimum, mid-point and maximum of the diameter range were optimised, and the same percentages were applied in every plot.

When the remaining basal area was used as

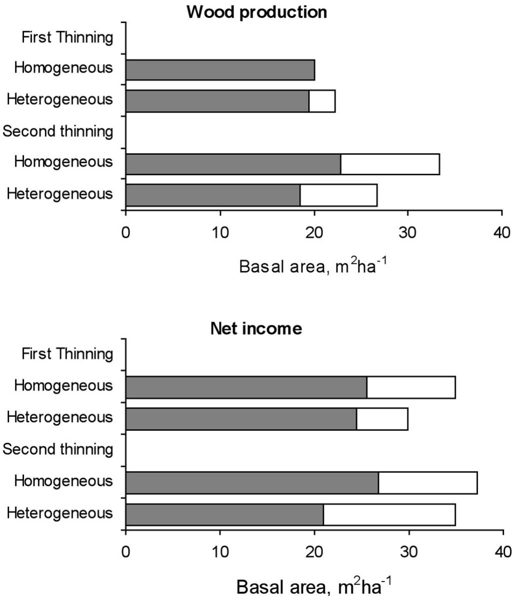

Fig. 7. Stand basal area before (total bar length) and after thinning (shaded part) in a homogeneous (variation multiplier 0) and heterogeneous (variation multiplier 1) pine stand when wood production or mean annual net income was maximized.

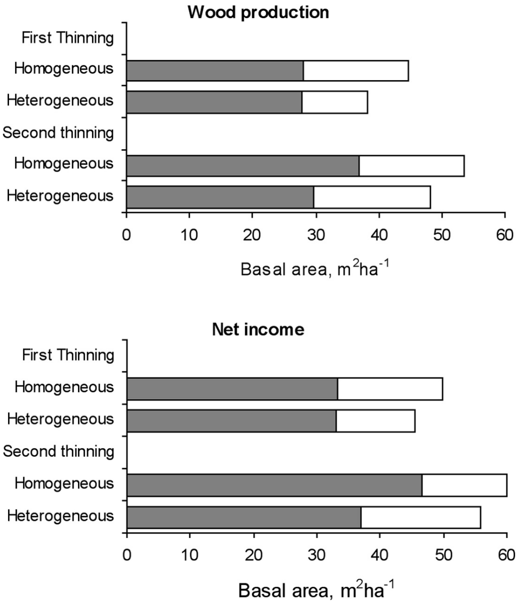

Fig. 8. Stand basal area before (total bar length) and after thinning (shaded part) in a homogeneous (variation multiplier 0) and heterogeneous (variation multiplier 1) spruce stand when wood production or mean annual net income was maximized

a decision variable in both thinnings the soil expectation values were lower than obtained when remaining basal area was used only in the first thinning (Table 4). The highest soil expectation values were reached when three harvest percentages were optimised in both thinnings. Only in a heterogeneous spruce stand our decision variables were equally good as the best inspected set. These results suggest that our primary set of decision variables was not optimal. Varying harvest percentages for different tree sizes allow thinnings from above to be used already in the first thinning, which yields higher soil expectation values than uniform thinnings. However, because it was assumed that a strict thinning from above cannot be implemented in the first thinning, due to the need for opening strip roads and removing low quality trees, we used the remaining basal area as the decision variable, and forced the thinning percentage to be the same in all diameter classes.

Table 4. Soil expectation value with 2% discounting rate (€ha^{-1}) when the thinning intensity is specified with remaining basal area in both thinnings (G-G), remaining basal area in the first thinning and three harvest percentages in the second thinning (G-H%) or three harvest percentages in both thinnings (H%-H%).

Stand	Variation multiplier	Decision variables for thinning intensity		
		G-G	G-H%	H%-H%
Pine				
– homogeneous	0	1414	1439	1598
– heterogeneous	1	1291	1356	1489
Spruce				
– homogeneous	0	3353	3455	3593
– heterogeneous	1	2714	2918	2918

4 Discussion

The results of the study support the hypothesis that the heterogeneity of a tree stand decreases its optimal thinning density. Also the remaining stand basal area should be lower in a heterogeneous stand, especially in spruce. In spruce the effect of stand heterogeneity at first commercial thinning could still be seen in the second thinning, which occurred at lower basal areas in heterogeneous stands. This kind of long-term effect was weaker in pine.

Increasing heterogeneity decreased the soil expectation value. The net income (in pine and spruce) and timber harvest (in spruce) also decreased when soil expectation value was maximised in increasingly heterogeneous stands. The same happened when wood production or net income was maximised. The results indicate that heterogeneity decreases stand productivity even when the first thinning aims at reducing variation in basal area between sub-areas of the stand.

Our results could be interpreted so that optimisations for homogeneous stands give misleading results for heterogeneous stands. The conclusion is the same for optimisations based on a single plot, or a single set of sample trees. If stands of variation multiplier equal to one are regarded as typical cases (our empirical data had a variation multiplier equal to one), the real optima for typical stands differ much from the optima based on a set of average stand characteristics. However, the interpretation is not so straightforward since the estimated within stand variation was based on plots (Sundström 2001, Anttila 2002) smaller than the ones that were used to develop the growth models (Hynynen et al. 2002). Small plot size results in high variation in stand characteristics and exaggerates the production differences in different parts of the stand.

The growth models used in the analysis were based on three-plot clusters, each plot having at least 35 to 40 trees (Hynynen et al. 2002). This means that a minimum of 100 to 120 trees per stand was measured, which corresponds to plots clearly larger than the ones used in this study to estimate the within stand variation. Therefore, a typical variation multiplier of a real stand, with a plot size similar as in the growth modelling data, may be clearly smaller than one. However, this fact does not invalidate our conclusions that optimal management depends on stand heterogeneity, and optimisation results obtained for homogeneous stands or a single plot are not directly applicable to heterogeneous stands.

The results about optimal stand management, if heterogeneity is not considered, are in several respects similar to many earlier studies (e.g. Valsta 1992a,b, Pukkala et al. 1998, Hyytiäinen and Tahvonen, 2002): increasing discounting rate shortens optimal rotation lengths (e.g. Hyytiäinen and Tahvonen 2002), and high thinnings should be conducted (Valsta 1992b, Pukkala and Miina 1998). In our stands, the optimal thinning type was almost invariably high thinning. In some cases it was optimal to thin from both ends from the diameter distribution (cf. Pukkala et al. 1998, Hyytiäinen et al. 2005). Medium-sized trees never had the highest harvest percentage.

All the results of this study are based on the assumption that the models and calculation techniques used in the analyses are correct. This assumption is certainly wrong since both the models and the calculation techniques simplify reality. However, the results make sense and strongly support the hypotheses, which were based on earlier literature.

References

Anttila, P. 2002. Updating stand level inventory data applying growth models and visual interpretation of aerial photographs. Silva Fennica 36(2): 549–560.

— , Haara, A., Maltamo, M., Miina, J. & Päivinen, R. 2001. Research data for forest mensuration. University of Joensuu, Faculty of Forestry. Research Notes 128. 9 p. + CD-rom. (In Finnish).

Deville, J-C. & Särndal, C-E. 1992. Calibration estimators in survey sampling. Journal of American Statistical Association 87: 376–382.

Hooke, R. & Jeeves, T.A. 1961. "Direct search" solution of numerical and statistical problems. Journal of the Association for Computing Machinery 8: 212–229.

Hynynen, J., Ojansuu, R., Hökkä, H., Siipilehto, J., Salminen, H. & Haapala, P. 2002. Models for predicting stand development in MELA system.

Finnish Forest Research Institute, Research Papers 835. 116 p.

Hyvän metsänhoidon suositukset 2001. Metsätalouden kehittämiskeskus Tapion julkaisuja 13/2001. 95 p. (In Finnish).

Hyytiäinen, K. 2004. Integrating economics and ecology in stand-level timber production. Finnish Forest Research Institute, Research Papers 908. 42 p. + app.

— & Tahvonen, O. 2002. Economics of forest thinnings and rotation periods for Finnish conifer cultures. Scandinavian Journal of Forest Research 11: 336–342.

— Tahvonen, O. & Valsta, L. 2005. Optimum juvenile density, harvesting, and stand structure in even-aged Scots pine stands. Forest Science 51(2): 120–133.

Kangas, A. & Maltamo, M. 2000. Calibrating predicted diameter distribution with additional information. Forest Science 46: 390–396.

Kilkki, P., Pohjola, T. & Pohtila, E. 1985. Use of the spatial distribution of trees in thinning models. Silva Fennica 19(2): 137–143. (In Finnish with English summary).

Laasasenaho, J. 1982. Taper curve and volume equations for pine spruce and birch. Communicationes Instituti Forestalis Fenniae 108.74 p.

Loetsch, F., Zöhrer, F. & Haller, K. 1973. Forest inventory. Vol. II. BLV Verlagsgesellschaft, München.

Monserud, R.A. 1989. Optimizing single-tree simulators. In: Burkhart, H.E., Rauscher, H.M. & Johann, K. (eds.). Artificial intelligence and growth models for forest management decisions. Sch. of For. & Wild. Resources, Virginia Poly. Inst. & St. Univ., Publ. No. FWS-1-89: 308–321.

Päivinen, R. 1980. Rinnankorkeusläpimittajakauman estimointi ja siihen perustuvia puustotunnusten estimointimenetelmiä. Licentiate thesis at University of Helsinki, Finland. 92 p. (In Finnish).

Pretzsch, H. 1995. On the effect of the spatial distribution of trees on stand growth. Allgemeine Forst- und Jagdzeitung 166(9/10): 190–201. (In German with English summary).

Pukkala, T. 1989. Predicting diameter growth in even-aged Scots pine stands with a spatial and non-spatial model. Silva Fennica 23(2): 101–116.

— 1990. A method for incorporating the within-stand variation into forest management planning. Scandinavian Journal of Forest Research 5: 263–275.

— & Kolström, T. 1991. Effect of spatial pattern of trees on the growth of a Norway spruce stand. A simulation model. Silva Fennica 25(3): 117–131.

— & Miina, J. 1998. Tree-selection algorithms for optimizing thinning using a distance-dependent growth model. Canadian Journal of Forest Research 28: 693–702.

— , Miina, J., Kurttila, M. & Kolström, T. 1998. A spatial yield model for optimizing the thinning regime of mixed stands of Pinus sylvestris and Picea abies. Scandinavian Journal of Forest Research 13: 31–42.

Rummukainen, A., Alanne, H. & Mikkonen, E. 1995. Wood procurement in the pressure of change – Resource evaluation model till year 2010. Acta Forestalia Fennica 248. 98 p.

Shao, G. & Shugart, H.H. 1997. A compatible growth-density stand model derived from a distance-dependent individual tree model. Forest Science 43(3): 443–446.

Siipilehto, J. 1999. Improving the accuracy of predicted basal-area diameter distribution in advanced stands by determining stem number. Silva Fennica 33(4): 281–301.

SPSS Inc. 2004. SPSS 11.5 Command Syntax Reference. SPSS Inc. Chicago, IL.

Sundström, E. 2001. Metsikkökuvion sisäisen vaihtelun huomioonottaminen metsäsuunnittelussa. Pro graduate thesis at the University of Joensuu, Finland. 39 p. + app. (In Finnish).

Valsta, L. 1992a. A scenario approach to stochastic anticipatory optimization in stand management. Forest Science 38: 430–447.

— 1992b. An optimization model for Norway spruce management based on individual-tree growth models. Acta Forestalia Fennica 232. 20 p.

Total of 28 references