DETERMINATION OF STEM VALUE

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A dynamic programming approach toward stem value estimation for standing Scots pine trees was developed. The determination of the saw log value was based on the sawing pattern and on the final products composition. The combination of taper curve models and bark models provided taper curves both over bark and under bark, which constituted the basis for the optimum stem scaling. A computer program was developed to determine the optimum log sequence of the stem aiming at maximizing the value of the final products. To examine the reliability of the computation system, 45 Scots pine sample trees from 29 stands were used as a test material. The stem values of sample trees were calculated in two ways: (1) with 12 measured diameters and (2) with 12 estimated diameters derived from measured tree characteristics. In both cases the values of the intermediate diameters were calculated via cubic spline interpolation.

1. INTRODUCTION

In theory, the amount of lumber and other final products should determine the stumpage price of the stem. More than 100 log rules have been developed in the past 150 years, and all purport to approximate the lumber yield of a log when its small end diameter and length are known (Hallock 1979).

The stumpage price of a stem is also affected by other factors, logging, transport, and manufacturing costs being the most important ones. If these additional costs can be regarded as constant per volume unit, then relative stem value can be determined as the sum of the values of the final products.

The determination of stem value can be approached as follows: (1) determine the unit prices of the final products and their composition for a given log, (2) determine the taper curves of the stem over and under bark and (3) determine the optimum scaling of the stem aiming at maximizing the total value of the final products.

For standing trees, taper curve models and bark models are needed. These models should give taper curves over and under bark as a function of measured standard tree characteristics.

Kikki and Varmola (1981) developed taper curve models for Scots pine. Päivinen (1978) developed models for bark thickness at breast height. Heikurainen (1982) prepared models for bark thickness at all heights of the stem. The application of these results gives the basis for determination of the stem value for a standing Scots pine.

This study attempts to develop an algorithm for maximizing the stem value by seeking out the optimum log sequence for the stem. The scaling is based on stem dimensions and it is assumed that no stem defects exist.
2. DETERMINATION OF LOG VALUE

The logical starting point in valuation of the stem is the determination of log value. This procedure calls for summing up all the sales values of the final products of the log. It is assumed that boards of equal thickness, slabs, sawdust, and bark are the "products" of a log.

The sawing pattern used in this study can be classified as live sawing (Richards et al. 1979). The following basic relations are in accordance with The International Log Rule devised by Clark in 1906 (see Avery 1967): the board thickness is 2.54 centimetres (1 inch) for each board-cut; 0.3175 centimetres (1/8 inch) will be lost in saw kerf and 0.1588 centimetres (1/16 inch) in shrinkage. Figure 1 presents the geometrical principle upon which the algorithm is based. The first board (opening face) is of minimum allowable width, 10 centimetres. The length A can be calculated as follows

\[ A = \sqrt{\frac{D}{2}} \left( \frac{W_{MIN}}{2} \right)^2, \]

where \( W_{MIN} \) = the minimum width of board
\( D \) = the diameter of the circle.

After the first board-cut

\[ A' = A - T, \]

where \( T \) = the total thickness of one board + one saw kerf + one shrinkage.

The board widths after the first board are

\[ WB = \sqrt{2D} - A', \]

where \( WB \) = The board width.

When the board-cut passes the circle center, the formula changes to

\[ A' = A + T. \]

Finally, the areas of cross cut sections of boards, saw keres, and shrinkages are added separately. The difference between the total area of the circle and the sum of these parts is the area of slabs. These results can be translated into percentages. Figure 2 presents the percentages of boards, slabs, sawdust, and shrinkage as a function of diameter of the circle, when \( D \) ranges from 100–500 millimetres. The smoothing curve has been made by hand.

If a log has a cylindrical form, the volumes of its products are reached by multiplying its total volume by the corresponding percentages. For a real log, however, allowance for taper should be made. Figure 3 shows the assumption concerning taper allowance. A sawlog is divided into one cylinder, some tubes and slabs. Lengths of tubes are equal to the lengths of boards fixed in advance. According to the top diameters of the cylinder and tubes, the percentages of the component products are derived from the smoothing curve. The volumes of different products are summed.

In order to estimate the bark volume of a log, the bark thickness at every point of the log must be known. The difference between the over bark and under bark log volume estimates equals the bark volume estimate.

The volumes of the log components are multiplied by their unit prices. The sales value of the log is:

\[ V = \sum V_i W_i, \]

where \( V_i \) = sales value of the log
\( V \) = volume of the \( i \)th component
\( W_i \) = unit price of the \( i \)th component.
3. OPTIMUM SCALING OF THE STEM

3.1. Determination of the taper curve

Log value information is utilized in stem valuation. For a felled tree, different alternatives of log sequences can be analysed, because a sufficient number of the stem diameters is easily measurable. For a standing tree, the taper curve estimation is the basic prerequisite for the optimization procedure.

Kilikki and Varmola (1981) have developed taper curve models for Scots pine. Using these taper curve models, 12 relative-height diameters over bark can be derived after a few measured variables such as height of the tree, diameter at breast height, diameter at 6 metres height, and crown height have been given as inputs. A natural cubic spline function is employed to interpolate the intermediate diameters; thereafter a continuous taper curve estimate of the stem over bark is available. Hekkunen (1982) has developed models for bark thickness at all heights of the stem. Bark thickness at breast height plus a few other tree characteristics are the inputs in his models. Päivinen (1978) developed models for bark thickness at breast height. Results from these two studies have been employed in the derivation of the under bark taper curve.

3.2. Stem value maximization

Optimum scaling of the stem is based on the principle of dynamic programming (see e.g. Hillier and Lieberman 1980; Strand 1987). A stem can be divided into m sawlogs, with \( m = 0, 1, 2, \ldots \). If there are \( n \) different standard lengths of sawlog, the core of the stem value maximization is to find out the optimum log sequence from a total of \( m^n \) theoretical log sequences. In practice the number of possible sawlog sequences is less than that figure.

Figure 4 illustrates the stem value maximization procedure in this study. The possible cutting points of the stem are located at 10 centimetre intervals. At an arbitrary point \( I \), the first possible arrangement is the optimum log sequence at point \( M_1 \), 10 centimetres backward to the bottom of the stem, plus one 10-centimetre-long pulpwod section if the diameter at point \( I \) is larger than that of equal to the minimum diameter of pulpwod logs. The value of the second arrangement at point \( I \) is equal to the maximum value at point \( M_1 \) plus the the second arrangement at point \( I \) is equal to the maximum value at point \( M_1 \) plus the value of that 4.0-metre-long sawlog. This recursive procedure continues until the last possible arrangement, the longest sawlog plus the optimum log sequence at point \( M_n \), is examined. Of all arrangements, the one having the largest value represents the optimum log sequence at point \( I \) and its value is the maximum value at point \( I \).

This maximizing routine starts from the stump height of the stem (1 percent of the tree height), is executed at every 10 centimetre interval and ends at the top of the stem.

4. COMPUTER ALGORITHM

The structure of the computer system developed in this study is illustrated in Figure 5. For a felled tree, measured values of 12 diameters over bark at 1, 5, 10, 15, 20, 30, 40, 50, 60, 70, 80 and 90 percent of total tree height are given as input. In order to estimate the taper curve of a standing tree, a taper curve model subroutine is introduced. The subroutine requires diameter at breast height and height of the tree as its input variables; provisory additional input variables are the diameter at 6 metres height and crown height.

In order to get estimates for the intermediate diameters, cubic splines with arbitrary second derivative end condition are employed for computing an interpolatory approximation to a given set of points (see program ICSICU, IMSL Library 2, 1977). In addition to the 12 relative-height diameters, three extra diameters are given for the cubic spline interpolation:

\[
D_{oa} = 5.0 \text{ mm} \\
D_{oa} = 2.0 D_{oa} - D_{oa} \\
D_{oa} = 2.1 D_{oa} - D_{oa}
\]

where \( D_{oa} \) = the diameter at relative height \( n \).

The condition to be fulfilled is that the diameter at all possible cutting points, given by all possible combinations of different standard lengths of logs, should be available. Since the difference between two successive standard sawlog lengths in this study is 30 centimetres, it was decided to calculate diameters at 10 centimetre intervals.

The input variables of the first bark model are diameter over bark at breast height, diameter over bark at 6 metres height, height of the tree, age of the tree, and effective temperature sum zone of the stand location (Päivinen 1978). The output variable is the double bark thickness at breast height.

The input variables of the second bark model are diameter over bark at breast height, diameter over bark at 6 metres height, height of the tree, age of the tree, crown length, and double bark thickness at breast height. The output variable is the double bark thickness at every 10 centimetre interval along the stem.

After the execution of these two bark models, taper curves over bark and under bark are ready.

Formula

\[
V = B_1 L
\]

where \( B_1 \) = the cross section area at the middle point of the stem section and \( L \) = the length of the stem section

is used to calculate the volumes of the 10-centimetre-long stem sections over bark and under bark. The difference between these two volumes is the bark volume.

The stem is divided into sawlog, pulpwod and waste wood parts. The minimum top diameter under bark of sawlogs is 160 millimetres, the minimum top diameter under bark of pulpwod log is 70 millimetres. For a pulpwod tree, the length of pulpwod log should be at least 2 metres. There is no limitation for pulpwod log length in a sawlog tree.
5. RESULTS AND DISCUSSION

5.1. Timber assortments and stem value

Taper curve model with given dbh and height was employed to calculate sample results. For each dbh-height combination 10 values of the diameter at 6 metres height were employed in order to avoid possible biases attributable to the nonlinear relationship between the stem dimensions and stem value (cf. Päivinen 1983). These values were picked systematically at equidistant intervals from the conditional distribution (d_i, | d, h). For each taper curve its relative frequency was employed as a weight.

An empirical formula derived by Kikki (1982) was employed to estimate the relative crown height:

\[ h_i/h = \exp(-1.151 - 0.001367d + 0.0699h - 0.0011h^2) \]

where: \( h_i \) = crown height

The tree age was derived as a function of the tree height from the yield table of regularly thinned pine forest growing on Vaccinium forest site type, according to Koivisto (1959) (Figure 6).

The unit price of boards was assumed to be 350 Fm/km² regardless of their lengths which range from 4.0 metres to 6.1 metres. The unit price of pulpwood was 150 Fm/km² as for the slabs. The unit price of sawdust was 60 Fm/km², and the unit price of bark 50 Fm/km². The volume contents of a stem are divided as follows:

- saw log volume (over bark)
- board volume
- slab volume
- sawdust volume
- shrinkage volume
- bark volume
- stem volume (over bark)
- pulpwood volume (over bark)
- wastewood volume (over bark)
- bark volume

Fig. 6 Age curve for Scots pine stand on Vaccinium site.
Tables 1, 2, 3, and 4 demonstrate some results of the stem scaling algorithm. Table 1 shows the estimate of stem volume over bark, table 2 the saw log percentage, and table 3 the board percentage as a function of the diameter at breast height and height of the tree. Table 4 presents the optimal top diameter and log length of the first log.

5.2. Reliability of the algorithm

In order to examine the reliability of the algorithm, 492 Scots pine sample trees were tested. The sample trees were from 29 stands located on mineral soils in Southern and Central Finland. The site types of the stands varied from Ousted-Myrtilus type to Calluna type (Cajander 1949), the majority of the stands being from Vaccinium type.

The stem values of these sample trees were calculated in the two ways described above: (1) 12 measured diameters are given; (2) measured tree characteristics (height of the tree, diameter at breast height, diameter at 6 metres height, crown height, and age of the tree) are given as inputs and 12 diameters are estimated. In the comparison of the results between these two approaches, deviation percentage was calculated as:

\[ p = \frac{\bar{v} - \bar{v}_c}{\bar{v}_c} \times 100 \]

where \( p \) = deviation percentage
\( \bar{v} \) = stem value calculated using 12 measured diameters
\( \bar{v}_c \) = stem value calculated using 12 estimated diameters

In order to avoid having 0 value as the divisor, 47 sample trees with \( \bar{v}_c = 0 \) were omitted, and the total amount of sample trees was reduced to 445.

When the unit price of boards is assumed to be 350 Fmk/m³, 2.3 times the unit price of pulpwood, the average standard error of the stem value estimate is 7.0 percent. If the unit price of boards is increased to 700 Fmk/m³, the standard error of stem value estimation is increased to 12.2 percent. In the extreme case, when the unit price of boards is equal to the price of pulpwood, the standard error is 5.3 percent, nearly equal to the standard error of stem volume estimation, which is 4.8 percent.

If only the expected value of \( d_b \) was employed and the unit price of boards is 350 Fmk/m³, the standard error of the stem value estimate is 12.2 percent and the bias 0.76 percent. (These figures correspond to the reliability of the stem value estimates given in section 5.1.)

The positive bias in the stem value estimates may be partly attributable to the relatively large errors at the stem diameter estimates. In order to avoid this bias, the average stem value based on 10 different \( d_b \) values was tested (see section 5.1). However, no significant reduction in the residuals was discernible.
Table 3. The board percentage of the stem volume as a function of $d_{1.3}$ and $h$, %.

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6. SUMMARY

A dynamic programming approach toward stem value estimation for standing Scots pine trees was developed. The determination of the saw log value was based on the sawing pattern and on the final product's composition. The combination of taper curve models and bark models provided taper curves both over bark and under bark, which constituted the basis for the optimum stem scaling. A computer program was developed to determine the optimum log sequence of the stem aiming at maximizing the value of the final products.

To examine the reliability of the computation system, 445 Scots pine sample trees from 29 stands were used as a test material. The stem values of sample trees were calculated in two ways: (1) with 12 measured diameters and (2) with 12 estimated diameters derived from measured tree characteristics. In both cases the values of the intermediate diameters were calculated via cubic spline interpolation.

When diameter at breast height, diameter at 6 metres height, height of the tree, crown height, and age of the tree were known, and the unit price of board was 700 Fmk/m³ (4.7 times the unit price of pulpwood) the average standard error of the stem value estimate was 12.2 percent, for unit price of boards 350 Fmk/m³ the error was 7.0 percent, and for unit price of boards 150 Fmk/m³ only 3.3 percent. Given only diameter at breast height and height of the tree as input variables, the average standard error of the stem value estimate was 12.2 percent when the unit price of boards was 350 Fmk/m³.
REFERENCES


