

Appendix 1

The locations of aggregation centres (in scenario 2) were simulated with varying intensity to create an aggregation of centres within each square. One point in each square was selected at random as the centre of aggregation; the density of aggregation centres (number ha⁻¹) decreased in the x and y-directions according to defined intensity functions. For the point with coordinates X and Y, the density of aggregation centres was given as $F(X,Y)=G(X,A)*G(Y,B)$ where

$$G(X) = \left(1 - \left(\frac{|X-WX|}{L \cdot XM}\right)^{POV}\right)^{(1/POV)} \quad \text{and} \quad G(Y) = \left(1 - \left(\frac{|Y-WY|}{L \cdot YM}\right)^{PNS}\right)^{\left(\frac{1}{PNS}\right)}. \quad (1)$$

Here, WX and WY are the x and y coordinates of the randomly selected point of maximum intensity and L is the lengths of the squares' sides. XM, YM, POV and PNS are input parameters that specify the rate at which density decreases from the point at WY and WX and thus define the degree of aggregation. In this study, XM and YM were set to 1.5 and POV and PNS were set to 0.4.

Appendix 2

The variance of the estimator of the mean number of damaged trees ha^{-1} across the entire study area based on a simple random sampling of a single year's panel is given as (Cochran 1977, p. 277)

$$V(\hat{Y}) = \frac{1}{n} \left((1-f)\sigma_b^2 + \frac{\sum_{i=1}^N V(\hat{y}_i)}{N} \right) \quad (1)$$

where f is the sample fraction n/N , $V(\hat{y}_i)$ is the variance of the estimator in square i and σ_b^2 is the between-square variance i.e. $\sigma_b^2 = (N-1)^{-1} \sum_{i=1}^N (y_i - \bar{Y})^2$. To calculate this variance for a study area with a mixture of squares with damaged trees and empty squares, we needed to express the variance of \hat{Y} in terms of the mean value and variance components for the subpopulation of squares containing damaged trees. Let N_d be the total number of squares containing damaged trees, P the proportion of squares with damaged trees (i.e., N_d/N), \bar{Y}_d the mean number of damaged trees ha^{-1} in the subpopulation and σ_d^2 the between square variance in the subpopulation. To express the between-square variance for the total study area, σ_b^2 , in terms of σ_d^2 , the summation in σ_b^2 can be expanded as

$$\begin{aligned} \sum_{i=1}^N (y_i - \bar{Y})^2 &= \sum_{i=1}^N y_i^2 - N\bar{Y}^2 = \sum_{i=1}^{N_d} y_i^2 - N_d \cdot \bar{Y}_d^2 + N_d \cdot \bar{Y}_d^2 - N\bar{Y}^2 \\ &= \sum_{i=1}^{N_d} (y_i - \bar{Y}_d)^2 + N_d \cdot \bar{Y}_d^2 - N \cdot \frac{N_d}{N^2} \bar{Y}_d^2 = \sum_{i=1}^{N_d} (y_i - \bar{Y}_d)^2 + \frac{(N - N_d)N_d}{N} \cdot \bar{Y}_d^2 \\ &= \sum_{i=1}^{N_d} (y_i - \bar{Y}_d)^2 + N_d(1 - P) \cdot \bar{Y}_d^2 \end{aligned} \quad (2)$$

Hence (see also Cochran, 1977, p. 38), the between square variance for all squares can be expressed in terms of the between square variance for the subpopulation of squares containing damaged trees as

$$(N-1)\sigma_b^2 = (N_d-1)\sigma_d^2 + N_d \cdot (1-P)\bar{Y}_d^2 \quad (3)$$

Following Cochran (1977 p. 38); if $1/N$ and $1/N_d$ are small this expression can be simplified to

$$\sigma_b^2 = P\sigma_d^2 + P(1-P)\bar{Y}_d^2 \quad (4)$$

Hence, the first component in eq. 1 can be expressed as

$$(1-f) \frac{1}{n} (P\sigma_d^2 + P(1-P)\bar{Y}_d^2). \quad (5)$$

The second component is given as

$$\frac{1}{n} \cdot P \cdot \frac{\sum_{i=1}^{N_d} V(\hat{y}_i)}{N_d} \quad (6)$$

since $V(\hat{y}_i)$ is zero in squares with no damaged trees.

By combining the two parts, the variance of \hat{Y} is given as

$$V(\hat{Y}) = \frac{P}{n} \left((1 - f)(\sigma_d^2 + (1 - P)\bar{Y}_d^2) + \frac{\sum_{i=1}^{N_d} V(\hat{y}_i)}{N_d} \right). \quad (7)$$

References

Cochran, W.G. (1977). *Sampling techniques*. New York. Wiley. ISBN 047116240X,9780471162407.