Appendix A

Definition of model inputs

Index sets

i is the index of a stand; i=1…l

j is the index of scenario chosen: T is the scenario in which silviculture is oriented towards timber production in all stands, PN is the scenario with a silviculture oriented towards pine nut production and MX is the scenario where the best silviculture (timber or pine nuts) is chosen in each stand.

k is the index of the prescription applied in each stand; k=1…m

Variables

x_{ijk} is the area harvested in a stand i, under a silviculture j, in time period(s) defined by prescription k.

z_{ijk} are binary auxiliary variables to force variables x_{ijk} to be greater than or equal to a_{min}.

n_{NPV} and p_{NPV} are the negative and positive deviation variables for the goal net present value.

n_{THV} and p_{THV} are the negative and positive deviation variables for the goal volume of timber harvested.

n_{YPN} and p_{YPN} are the negative and positive deviation variables for the goal yield of pine nuts.

n_qH and p_qH are the negative and positive deviation variables for the goal volume control.

n_ra and p_ra are the negative and positive deviation variables for the goal area control.

n_sI and p_sI are the negative and positive deviation variables for the goal ending forest inventory

D represents the maximum deviation between a goal and its aspiration level.

λ indicates the goal programming model chosen, or an intermediate solution

Coefficients

npv_{ijk} is the net present value per hectare harvested in stand *i*, under a silviculture *j* and prescription *k*.

npv_{ijk}^{T} is the net present value of timber harvests per hectare harvested in stand *i*, under a silviculture *j* and prescription *k*.

npv_{ijk}^{PN} is the net present value of pine nut yield per hectare in stand *i*, under a silviculture *j* and prescription *k*.

th_{ijk} : is the volume per hectare harvested in stand *i*, under a silviculture *j* and prescription *k*.

ypn_{ijk} is the sum of annual yields of pine nuts in a stand *i*, under a silviculture *j*, in the periods of the planning horizon defined by prescription *k*.

ypn_{ijkd} is the yield of pine nuts in a stand *i*, under a silviculture *j*, in the periods of the planning horizon defined by prescription *k* in a year *d*.

Accounting variables

NPV: is the total net present value of timber harvests and pine nut yield during the planning horizon

NPV^{T} : is the net present value of timber harvests during the planning horizon

NPV^{PN} : is the net present value of pine nuts yield during the planning horizon

TH: is the timber harvested during the planning horizon

YPN: is the yield of pine nuts harvested during the planning horizon

H_q: is the timber harvested during the period *q*

A_r: area by each age class *r*

V_s^f : ending forest inventory for all the stands belonging to site index *s*

Constants

TA: total forest area

a_i: is the area existing initially in stand *i*

q: time period into which the planning horizon is divided

Q: number of time periods in the planning horizon (1,…*q*,…*Q*)

r: age class (comprised at least two time periods)

V_s^i : initial forest inventory for all the stands belonging to site index *s*

d: each year of the planning horizon

a_{min}: is the minimum harvest area (3ha in this case study).

Z: is the total area (*TA*) divided by the number of age classes (6).

LP MODEL

Objective functions

Maximize net present value associated with timber: Max NPV^T :

$$NPV^T = \sum_{i=1}^{l} \sum_{j=T}^{PN} \sum_{k=1}^{m} npv_{ijk}^T \cdot x_{ijk} \quad (1)$$

Maximize net present value associated with pine nuts: Max NPV^{PN} :

$$NPV^{PN} = \sum_{i=1}^{l} \sum_{j=T}^{PN} \sum_{k=1}^{m} npv_{ijk}^{PN} \cdot x_{ijk} \quad (2)$$

Maximize total net present value NPV :

$$NPV = \sum_{i=1}^{l} \sum_{j=T}^{PN} \sum_{k=1}^{m} npv_{ijk} \cdot x_{ijk} \quad (3)$$

Max volume of timber harvested TH :

$$TH = \sum_{i=1}^{l} \sum_{j=T}^{PN} \sum_{k=1}^{m} th_{ijk} \cdot x_{ijk} \quad (4)$$

Max yield of pine nuts YPN:

$$YPN = \sum_{i=1}^{l} \sum_{j=T}^{PN} \sum_{k=1}^{m} ypn_{ijk} \cdot x_{ijk} \quad (5)$$

Endogenous constraints

Area availability constraints:

$$\sum_{i=1}^{l} \sum_{j=T}^{PN} \sum_{k=1}^{m} x_{ijk} \leq a_i \quad \forall i \quad (6)$$

Exogenous constraints

Volume control:

$$H_q = \sum_{i=1}^{l} \sum_{j=T}^{PN} thv_{ijq} \cdot x_{ijq} \quad q = 1,\ldots 9 \quad (7)$$

$$H_q - H_{q+1} = 0 \quad (8)$$

Area control:

For age classes lesser than or equal to 100 years

$$A_r = \sum_{i=1}^{l} \sum_{j=T}^{PN} \sum_{q=2r-1}^{2r} x_{ijq} \quad r = 1,\ldots 5 \quad (9)$$

For age class greater than 100 years (no final cuttings in the planning horizon)

$$A_r = \sum_{i=1}^{l} \sum_{j=T}^{PN} \sum_{q>2r} x_{ijq} \quad r > 5 \quad (10)$$

$$A_r = Z \quad \forall r \quad (11)$$

91 Ending forest inventory:

92
$$V_s^f = \sum_{i=1}^{l}\sum_{j=T}^{PN}\sum_{k=1}^{m} v_{ijks}^f \cdot x_{ijks} \quad \forall s$$
$$V_s^f = V_s^i \qquad (12)$$

93

94 Minimum harvest area:

95
$$x_{ijkq} - a_{\min} z_{ijkq} \geq 0 \quad \forall i,j,k,q$$
$$x_{ijkq} - TA z_{ijkq} \leq 0 \quad \forall i,j,k,q$$
$$z_{ijkq} \in \{0,1\} \quad \forall i,j,k,q \qquad (13)$$

96

97 Minimum pine nut yield in each year:

98
$$ypn_d = \sum_{i=1}^{l}\sum_{j=T}^{N}\sum_{k=1}^{m} ypn_{ijkd} \quad \forall d$$
$$ypn_d \geq 140000 \qquad (14)$$

99

100 GP MODEL

101 *Goals (for each scenario)*

102 Total net present value NPV:

103
$$NPV + n_{NPV} - p_{NPV} = NPV^* \qquad (15)$$

104 Volume of timber harvested THV:

105
$$THV + n_{THV} - p_{THV} = THV^* \qquad (16)$$

106 Yield of pine nuts YPN:

107
$$YPN + n_{YPN} - p_{YPN} = YPN^* \qquad (17)$$

108 Volume control:

109
$$H_{q+1} + n_q H - p_q H = H_q \quad q = 1,\ldots,Q-1 \qquad (18)$$

Area control

$$H_{q+1} + n_q H - p_q H = H_q \quad q = 1,\ldots,Q-1 \tag{19}$$

Ending forest inventory:

$$V_s^f + n_s I - p_s I = V_s^i \quad \forall s \tag{20}$$

where NPV^*, THV^*, YPN^* are the ideal values of the objectives *NPV, THV, YPN*

Achievement function

$$Min(1-\lambda)D + \lambda \left\{ \begin{array}{l} (n_{NPV} + p_{NPV}) + (n_{THV} + p_{THV}) + (n_{YPN} + p_{YPN}) + \\ \sum_{q=1}^{q=Q-1} (n_q H + p_q H) + \sum_{\forall r} (n_r a + p_r a) + \sum_{\forall s} (n_s I + p_s I) \end{array} \right\} \tag{21}$$